

What skills must a Grade 7 have in order to be well placed to make a success of High School Maths?

by Paul de Wet



About eleven years back when I was on the Maths staff at Hilton College, each of our Maths teachers visited a different feeder school to get a sense of what they were doing in their Maths Classrooms and to share some ideas about our expectations (mathematically) of the pupils they were sending on to us. This proved to be a very interesting and valuable exercise. There was considerable variation in the approach and in the scope of work being taught at the five schools visited. More recently, in October 2007, my friend and Headmaster of Felixton College, Ken Krige, asked me to talk to primary school teachers from a variety of Zululand prep schools on the skills required for High School Maths by a primary school leaver. My Head of Department here at Michaelhouse, Alan Adlington-Corfield and I decided to survey our Grade 8's and 9's at that time to get their take on the effectiveness of their primary school preparation for High School Maths. The results of this survey were also interesting and, together with our experience, they informed the talk that I subsequently gave at Felixton. Lastly, I recently read an excellent book entitled, "Where do I put the

decimal point?" by Elizabeth Ruedy. It is a must read for all Maths teachers as it explores Math Phobia and gives great tips for overcoming it. It too has been very useful in informing my thinking. I do not want this article to come across as prescriptive but rather hope that it will stimulate thought and discussion. If it adds any value to primary school teachers preparing pupils for High School Maths then it will have served its purpose well.

Most FET subjects have a two or three year curriculum. Pupils can even pick up some subjects late and still excel in them. A weakness in one area is unlikely to provide a handicap in another. For example, if one underperforms in Climatology in the Geography syllabus, one may still enjoy success in Urban Geography. Maths is considerably different in that every topic requires prior knowledge and the entire syllabus is integrated and dependent on sound algebraic skills and a feel for number. In a very real sense Maths has a 12 year syllabus. One can liken this to building a house over a 12 year period. Many pupils struggle in the final stages but this is almost always due to problems with the foundations. This further strengthens the case for good interaction between Maths teachers in the different phases of schooling.

One of the first things we discovered in our research was that our pupils' Maths marks were 6% lower in June of Grade 8 than at the end of Grade 7. In spite of this, a vast majority (84%) felt that they had been well prepared for Grade 8 Maths. Given that High School Maths involves the introduction of abstract concepts like Algebra while Primary school maths deals largely with the concrete concept of number, it is not altogether surprising that there is a fall off in marks. The ability to deal with number is a necessary but not sufficient skill for success in Algebra. Often this trend of "diminishing returns" continues through their High School Maths career with the marks deteriorating steadily. This is typically aggravated by a waning of confidence and a dislike for Maths although it is also ascribable to an increase in the volume and the complexity of the work. Irrespective of the reasons, it is hard to help a parent understand how his / her son or daughter has gone from an "A" in primary school to struggling to pass in Grade 12! I suspect that many primary schools' Maths marks are somewhat inflated.

For me the single most important attributes I would like to see in a new Grade 8 would be a confidence in, and an enjoyment of, Maths.

For this reason I do not take issue with pupils enjoying early success in Maths even if it means making the earlier work easy. Too many pupils enter high school already “bruised” and scared of Maths, believing that they “*cannot do it*”. I fear that their lack of confidence and enjoyment is often further undermined by their parents who say, “*Never mind, I could never do Maths either!*” Parents need to be told early on that this is not a helpful sentiment to express to their children!

The actual scope of Mathematics required by a Grade 8 is quite limited and could be summarised in one phrase as “*having a feel for number*”. High school teachers do not require pupils to come to them with any knowledge of algebra or negative numbers. In fact, most High School teachers would prefer to be able to introduce these topics from scratch. I fear that some Primary School teachers “accelerate” their pupils, introducing Grade 8 topics, in a noble yet misguided attempt to prepare them for high school. They would be better advised to spend the time building confidence in number.

I will identify and elaborate on some key skills which should form the foundation of a successful Maths career. Perhaps not surprisingly, many of these are not related specifically to Maths but are more general in nature:

An ability to read

Many pupils struggle with Maths because of an inability to read and comprehend a question, even one which does not involve terminology which is specific to Maths. For this reason, it is a great irony to me that the new Maths Literacy syllabus requires so much reading of pupils, many of whom struggle with reading.

An ability to follow instructions – verbal and written

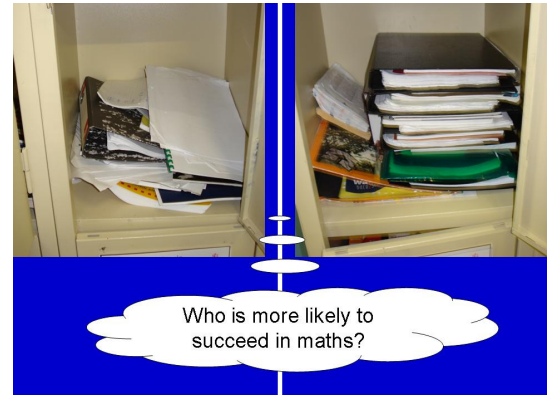
As simple as this one sounds, it is of paramount importance. Many pupils are fine with verbal instructions particularly when these are repeated and deconstructed into bite size chunks! They are typically less able to deal with written instructions which are not accompanied by verbal clarification. I would strongly encourage teachers at all levels to issue more written instructions. The ability to read a set of instructions and translate them into action is a key life skill.

An ability to work in groups

Our education model has historically undermined group work. The emphasis has been on individual achievement rather than group success. We still find that pupils are reluctant to work in groups. Quite apart from the fact that the ability to work in groups is a real life skill, I believe that there is opportunity to make Maths less threatening by encouraging group work and peer tutoring. I hope that all Maths teachers at all levels may be encouraged to try more group work in their classrooms.

Good organizational skills

The incremental nature of Maths means that it is vital that pupils are able to refer back to previous sections. Many pupils find it very difficult to keep their Maths files organized. Not surprisingly there is a strong correlation between those that manage their files well and those that enjoy success. One could argue that this is because clever pupils are better able to file but I would not discount the fact that a pupil who has a well organized file is more likely to enjoy success. I suspect that many primary schools use books for Maths but I would encourage Grade 7 teachers to force and teach the use of a file in preparation for High School.



An ability to communicate one's thinking clearly – both verbally and in writing

Many pupils know the answer but are not able to explain how they arrived at it. In order to qualify for part marks or method marks, pupils need to be able to communicate not just their answer but also their thinking which lead to it. Pupils really need to be given the time and space to **learn** to communicate their thinking. This is often enhanced or jeopardized by the classroom atmosphere. Pupils need to feel that they can express their thinking without a fear of being ridiculed in any way.

An ability to set work out neatly

Maths is a discipline and a subject which demands rigor. Historically the paradigm was one of a single correct method and a prescribed layout. Happily we have moved to greater flexibility, often encouraging pupils to come up with their own methods and layout. However, we need to be careful that we do not “*throw out the baby with the bathwater*” in the sense of allowing sloppy, unclear solutions or solutions which, despite leading to a correct answer, are mathematically correct en-route. Pupils should be encouraged to work down the page with one line of working per line. If one wishes to save paper then by all means separate the page into two columns but never work across the page. Good presentation is a habit which should be developed from the outset.

In the example below, the pupil has obtained the right answer but it is very difficult to work out what he has done! A better solution is given on the right.

d. $\frac{2}{3} + \frac{1}{2}$

$\frac{5}{7} - \frac{1}{2}$

$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6}$

$\frac{5}{7} - \frac{1}{2} = \frac{10}{14} - \frac{7}{14} = \frac{3}{14}$

$\frac{7}{6} \div \frac{3}{14} = \frac{7}{6} \times \frac{14}{3} = \frac{49}{9}$

$\frac{49}{9}$

careful! see me!

$\frac{2}{3} + \frac{1}{2}$

$\frac{5}{7} - \frac{1}{2}$

$\frac{4}{6} + \frac{3}{6}$

$\frac{10}{14} - \frac{7}{14}$

$\frac{7}{6}$

$\frac{3}{14}$

$\frac{7}{6} \times \frac{14}{3}$

$\frac{49}{9}$ or $5\frac{4}{9}$

Tenacity

Teaching maths is a balancing act between making pupils confident on the one hand but making them realise that it is a discipline that demands practice and tenacity on the other. One of my biggest frustrations is when pupils come and ask for help before even trying a problem. Teachers need to guard against merely giving the solution but should rather encourage pupils to make an effort to get started, offering a hint if necessary.

An ability to think laterally to cope with unfamiliar situations

Maths requires pupils to be able to apply their familiar knowledge to unfamiliar situations. Many people believe that lateral thinking and problem solving are innate skills. However, there is no doubt that one can improve one's problem solving ability with practice. Problems should be in context rather than in isolation and one cannot introduce problem solving at too young an age.

An appreciation of Maths as a language

Pupils need to understand that Maths is a language used to represent situations. Grade 8 pupils need to be familiar with all the terminology encountered at primary school. I am often amazed that pupils do not know the difference between a factor and a multiple. I find it helps to ask pupils to use the word in a sentence that illustrates their understanding of the concept. For example, “*3 is a factor of 12 since 3 divides exactly into 12*”. I think it is a good idea for all pupils, irrespective of Grade, to keep a Maths Dictionary in which they can record all new terms and their meanings.

Sound tables and bonds and strong numeracy skills

A couple of years back I was horrified to discover that my Grade 8 class averaged 70% on a basic test involving nothing more than tables (up to 12 times) and bonds. Furthermore, there was a strong correlation between the test results and the pupils’ performance in the algebra we were studying at the time.

One of the building blocks in High School Maths is factorizing. When one factorizes, for example:

$$x^2 + 11x + 30$$

to

$$(x + __)(x + __)$$

One now needs factors of 30 (numbers that multiply to give 30) which add to 11. Pupils without good tables and bonds often struggle for ages to discover 5 and 6 as the necessary numbers. I fear that the calculator as well as a changing emphasis in primary school maths has undermined the importance of tables and bonds.

Pupils should have a “feel” for number and for the relative size of a number. This enables them to test their solution against their intuition. Some years back I asked pupils to work out how many times a bicycle wheel of a given diameter would rotate on a 200 km ride. I was amazed at the number of pupils who, due to errors in converting units, arrived at answers like, 7! I was even more amazed that they did not stop to consider whether that made any sense in the context of the problem! Estimation is a very useful life skill. Try to encourage your pupils to estimate answers before they even start solving a problem.

An ability to understand that the same thing can be represented in different ways

I am often amused to hear a pupil say, “I like fractions but I cannot do percentages and decimals”. This suggests that pupils don’t really understand that all of these are simply different ways of representing numbers. Pupils should be comfortable in their ability to manipulate all of these and to compare them. A pupil with a feel for number will be comfortable listing the following set of numbers in ascending order:

23% ; $\frac{3}{8}$; 0.21 and $\frac{4}{9}$. I like to stress that % is the international standard for fractions, a kind of SI unit.

An understanding that the equals sign is sacred!

The equals sign is often abused by pupils making statements that make no sense mathematically even when the pupil ends up with the right answer. Consider the following example:

$$3 + 5 \times 2 = 10 = 13$$

A mathematically correct version is

$$\begin{aligned} &3 + 5 \times 2 \\ &= 3 + 10 \\ &= \underline{13} \rightarrow \end{aligned}$$

An understanding of what one is doing

There is no substitution for understanding in Maths. Many pupils will be able to apply algorithms (e.g. long multiplication, long division, addition of fractions) without really understanding what they are doing or why the algorithm works. Wherever possible pupils should be taught maths with understanding, rather than as a system of “black box” procedures. Pupils who understand what they are doing are more likely to be able to transfer their knowledge to other areas and are certainly more likely to be able to generalize a situation – one of the goals of algebra. Consider the following two examples:

$$\begin{aligned} &\frac{1}{3} + \frac{2}{5} \\ &= \frac{5}{15} + \frac{6}{15} \\ &= \frac{11}{15} \end{aligned} \rightarrow$$

Most pupils are able to add fractions but do they really understand what they are doing. I like to use an analogue of money. Suppose I have \$1 and you have R2. Before we can add them we need to get them into equivalent types. Here again an understanding of terminology helps. The denominator of a fraction **nominates** what type of fraction we are dealing with while the numerator **numerates** (counts) how many we have. Fractions are completely intuitive. Pupils who have had the time to play with a fraction kit constructing fractions will have no fear of them. Primary pupils who fully understand fractions will have no problem moving from $\frac{1}{3} + \frac{2}{5}$ to $\frac{1}{x} + \frac{a}{y}$ when they encounter it at high school.

$$\begin{array}{r} 253 \\ \times 17 \\ \hline 2530 \\ 1771 \\ \hline 4301 \end{array}$$

Most pupils will probably be able to do this sum but I fear many will simply be following a set of steps without wondering why or how it works.

I hope that pupils will understand that the above sum can be done as follows:

$$\begin{aligned} &253 \times 17 \\ &= 253 \times (10 + 7) \\ &= 253 \times 10 + 253 \times 7 \\ &= 2530 + 1771 \\ &= 4301 \end{aligned}$$

This helps to see why the algorithm works. It also reinforces the distributive law in a non-threatening way. Pupils who understand why $253 \times (10 + 7) = 253 \times 10 + 253 \times 7$ will have no problem understanding why $2(x + ay) = 2x + 2ay$ when they encounter it at high school.

An ability to look for patterns or rules

Mathematics is all about looking for patterns. While primary school pupils will not have the language of algebra to describe patterns, they certainly should be able to describe them in words.

For example, a pupil who can complete the following table:

1	2	3	4	5	6	7	10	
5	8	11	14	17				302

will likely be able to describe the rule in words, viz. “multiply the top number by 3 and then add 2” It is a short trip to take this to $3n + 2$ when algebra is introduced in Grade 8. The “backwards working” required to complete the following column is requiring pupils to solve the equation $3 \times \square + 2 = 302$

A pupil who can do this understands the order of operations (BOMDAS) required to construct an expression as well as the reverse order of operations (SADMOB) required to deconstruct one.

Conclusion

In summary, pupils need to leave primary school with a real feel for number, a confidence in their ability and an enjoyment of Maths as a human endeavour. I would welcome comment on this article and would be very happy to share further ideas with interested primary school Maths teachers. E-mail me on paudew@michaelhouse.org