SHAYA IZIBALO MATHS INSTITUTE in partnership with KWAZULU-NATAL PROVINCE EDUCATION REPUBLIC OF SOUTH AFRICA Supported by

Probability and Counting Techniques Workshop – Monday 28 July 2025

1. A die is thrown and a card is drawn from a pack. Determine the following probabilities:

a. $P(3$ on die $\cap$ a red card) (2)

$= P(3$ on die$) \times$ P(red card) since they are independent!

$= \frac{1}{6} \times \frac{1}{2}$

$= \frac{1}{12}$

b. P(prime on die $\cap$ a heart) (2)

=P(prime on die) $\times$ $P(a$ heart$)$ since they are independent

$= \frac{3}{6} \times \frac{1}{4}$

$= \frac{1}{8}$

c. P(factor of 9 on die $\cup$ a black card) (3)

$= P(\text{factor of } 9) + P(\text{black card}) - P(\text{factor of } 9) \times P(\text{black card})$

$= \frac{2}{6} + \frac{1}{2} - \frac{2}{6} \times \frac{1}{2}$

$= \frac{2}{3}$

d. P(even on die $\cup$ a spade) (3)

$= P(\text{even on die}) + P(\text{spade}) - P(\text{even on die}) \times P(\text{spade})$

$= \frac{1}{6} + \frac{1}{4} - \frac{1}{6} \times \frac{1}{4}$

$= \frac{3}{8}$

2. Two friends are comparing their chances of getting a ticket to a certain concert. Suppose Thandi has a 40% chance, Bongi has a 16% chance and there is 6,4% chance that both get a ticket.

Are their chances of getting a ticket:

a. mutually exclusive? Justify. (2)

No, since $P(\text{both getting a ticket}) \neq 0$

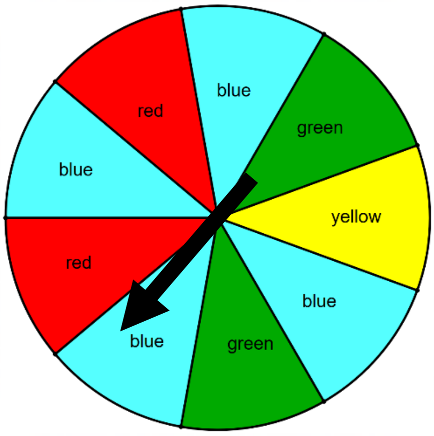
b. Independent? Justify. (2)

Yes, since P(Bongi $\cap$ $Thandi) = 0{,}064$ and $P(Bongi) \times P(Thandi) = 0{,}064$

3. Suppose event A is six times more likely to happen than event B and that they are Mutually exclusive. If $P(A \cup B) = 0{,}84$ then determine $P(B)$ (4)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0{,}84 = P(A) + P(B) - 0$$
$$\text{but } P(A) = 6P(B)$$
$$\therefore 0{,}84 = 7P(B)$$
$$P(B) = 0{,}12$$

4. I spin the spinner below. It has nine equally-sized sectors.



Determine:

a. P(red')
$$= \frac{2}{9}$$

b. P(green ∪ blue)
$$= \frac{6}{9} = \frac{2}{3}$$

c. $P(2\ \text{greens in 2 spins})$
$$= P(\text{green}) \times P(\text{green}) \qquad \text{they are independent}$$
$$= \frac{2}{9} \times \frac{2}{9}$$
$$= \frac{4}{81}$$

d. $P(1\ \text{red in 2 spins})$
$$= P(red) \times P(red') + P(red') \times P(red)$$
$$= \frac{2}{9} \times \frac{7}{9} + \frac{7}{9} \times \frac{2}{9}$$
$$= \frac{28}{81}$$

e. P(at least one blue in 5 spins)
$$= 1 - P(no\ \text{blues in 5 spins})$$
$$= 1 - \left(\frac{5}{9}\right)^5$$
$$= 0{,}947 \text{ or } 94{,}7\%$$

f. The minimum number of times I must spin it to be at least 90% sure of getting at least one yellow.

$P(\text{at least 1 yellow}) = 1 - P(\text{no yellows})$

$1 - \left(\frac{8}{9}\right)^n = 0{,}9$

$0{,}1 = \left(\frac{8}{9}\right)^n$

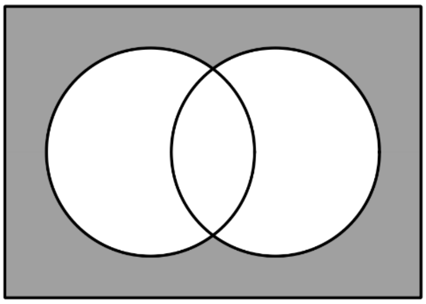
$n = \log_{\frac{8}{9}} 0{,}1$

$n = 19{,}55$

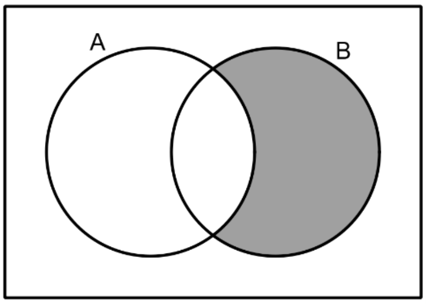
so, we must spin it 20 times to be at least 90% sure of at least one yellow

5. Shade the requested areas on the venn diagrams below: (8)

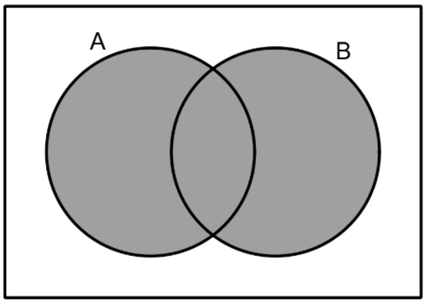
$A' \cap B'$



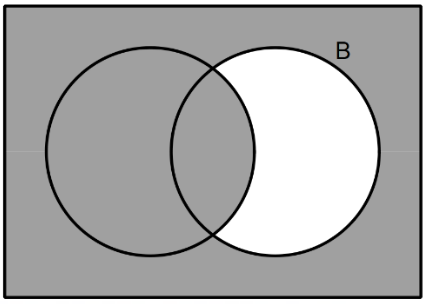
$A' \cap B$



$A \cup B$



$A \cup B'$



6. The probability that Phumlani finishes the Comrades is 0,8 while the probability that Sanele finishes is 0,7. If the probability that either Phumlani or Sanele finishes is 94% then determine the probability that both finish and determine whether or not their chances of finishing are independent or not. (4)

$P(\text{Phumlani} \cup \text{Sanele}) = P(\text{Phumlani}) + P(\text{Sanele}) - P(\text{Phumlani} \cap \text{Sanele})$

$0{,}94 = 0{,}7 + 0{,}8 - P(\text{Phumlani} \cap \text{Sanele})$

$0{,}94 = 1{,}5 - P(\text{Phumlani} \cap \text{Sanele})$

$P(\text{Phumlani} \cap \text{Sanele}) = 1{,}5 - 0{,}94$

$P(\text{Phumlani} \cap \text{Sanele}) = 0{,}56$

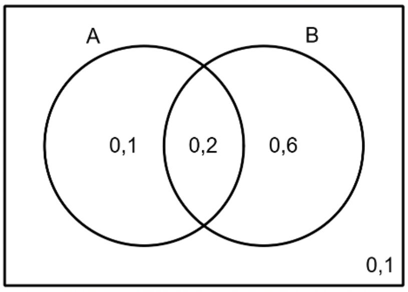
Now $P(\text{Phumlani}) \times P(\text{Sanele}) = 0{,}7 \times 0{,}8 = 0{,}56$

Their chances of finishing ARE independent

7. Given:

$P(A' \cap B) = 0{,}6$ $\quad\quad$ $P(A \cap B) = 0{,}2$ $\quad\quad$ $P(A) = 0{,}3$

Draw a venn diagram. (3)



Now use it to determine:

a. If A and B are mutually exclusive? Justify your answer. (1)

no, since $P(A \cap B) \neq 0$

b. If A and B are independent? Justify your answer. (2)

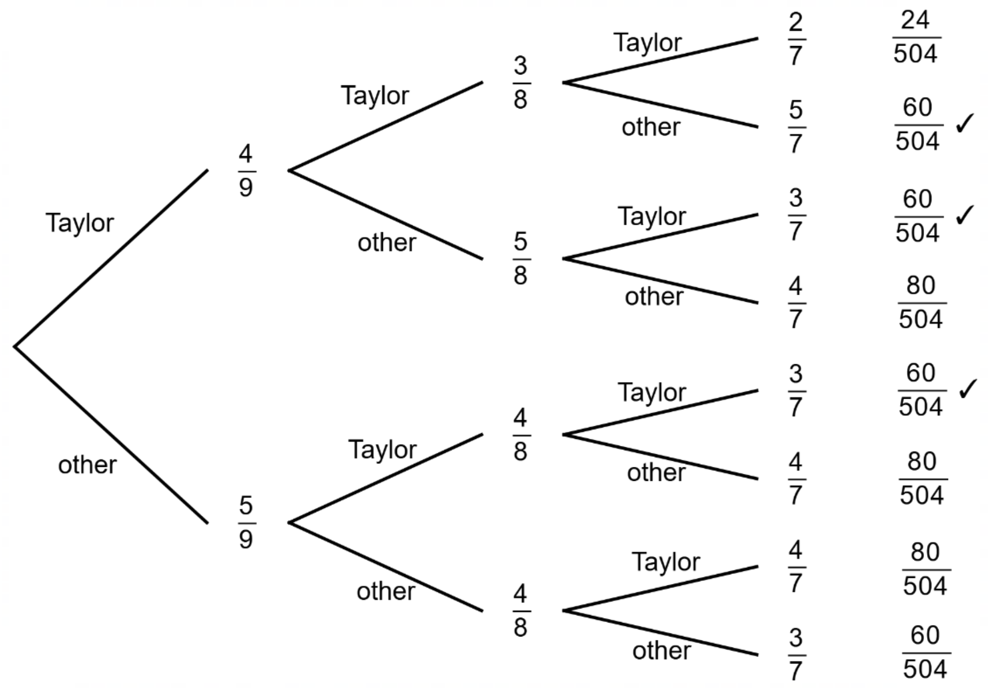
No, since $P(A) \times P(B) = 0{,}3 \times 0{,}8 = 0{,}24$
but $P(A \cap B) = 0{,}2$

c. $P(A' \cap B')$ (1)
$= 0{,}1$

d) $P(A \cup B)$ (1)
$= 0{,}9$

8. A music play list on my phone has 9 songs. It plays them in random order and will not replay a song until all the songs have played. Four of the songs are by Taylor Swift.

Draw a tree diagram and use it to determine the probability that exactly 2 of the next three songs are by Taylor Swift. (5)

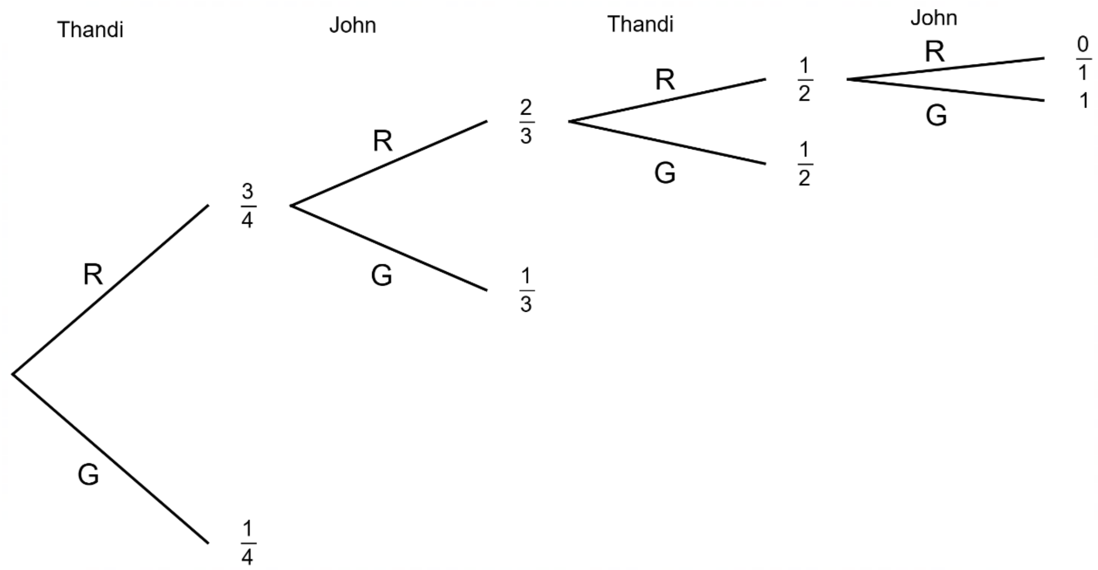


$$P(\text{exactly 2 of next 3 by } Taylor) = \frac{60}{504} + \frac{60}{504} + \frac{60}{504} = \frac{180}{504} = 35{,}71\%$$

8. A bag contains 1 green bead and 3 red beads. John and Thandi take turns **removing** A bead from the bag with Thandi going first. The person who draws the green bead wins.

Determine the probability that Thandi wins.

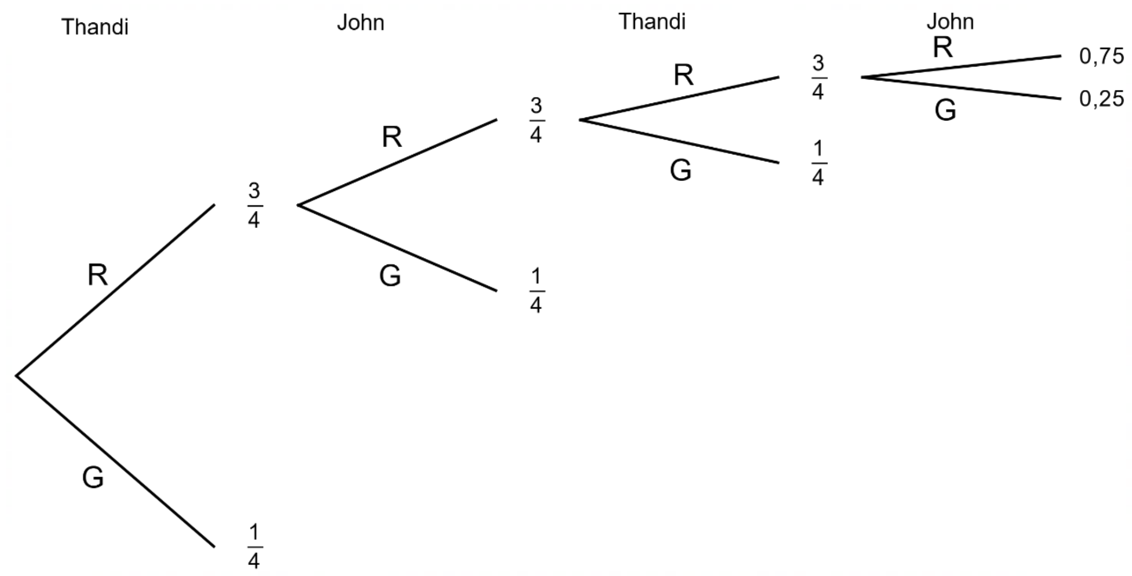
Hint: use a tree diagram.



$$\text{Thandi's chance of } winning = \frac{1}{4} + \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2}$$

$$\text{Thandi's chance of } winning = 0{,}5$$

9. Repeat question 8 but this time with the bead being **replaced** after each turn. (6)



Thandi's chance of winning:

$$\frac{1}{4}+\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)+\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)+\left(\frac{3}{4}\right)^6\left(\frac{1}{4}\right)+\ldots.$$

this is a convergent geometric series with $a=\frac{1}{4}$ and $r=\left(\frac{3}{4}\right)^2$

$$S_\infty=\frac{a}{1-r}$$

$$S_\infty=\frac{\frac{1}{4}}{1-\left(\frac{3}{4}\right)^2}$$

$$\underline{S_\infty=57{,}14\%}$$

Just as a check....

John's chance of winning:

$$\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)+\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)+\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)+\ldots\ldots$$

$$S_\infty=\frac{\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$$

$$\underline{S_\infty=42{,}86}$$

10. A computer code takes the form of 3 digits which may not be repeated followed by 2 letters which may be repeated.

An example of such a code is 015AA

a. How many such codes are possible? (3)

$10 \times 9 \times 8 \times 26 \times 26 = 486\ 720$ codes are possible

b. Suppose that twenty billion (20 000 000 000) unique codes are needed. It is decided to add extra letters. **Calculate** the **minimum** number of extra letters which will be needed. (5)

If we have a total of n letters then we have:

$10 \times 9 \times 8 \times 26^n$ possible codes and we need:

$10 \times 9 \times 8 \times 26^n = 20\ 000\ 000\ 000$

$$26^n = \frac{20\ 000\ 000\ 000}{10 \times 9 \times 8}$$

$$n = \log_{26}\left(\frac{20\ 000\ 000\ 000}{10 \times 9 \times 8}\right)$$

$n = 5{,}26066269$

We need a total of 6 letters so we must add 4 more

11. Adam is forming 5 digit numbers by choosing from the digits

2 ; 3 ; 4 ; 6 ; 7 ; 8 and 9

There is only one of each of the above digits.

a. How many numbers can Adam form? (2)

$7 \times 6 \times 5 \times 4 \times 3 = 2\,520$ numbers are possible

b. If one of the numbers formed by Adam is chosen at random what is the probability that it is even AND exceeds 70 000? (5)

Ends in 2
must start with 7 or 8 or 9
$3 \times 5 \times 4 \times 3$
$= 180$ possibilities

Ends in 4
must start with 7 or 8 or 9
$3 \times 5 \times 4 \times 3$
$= 180$ possibilities

Ends in 6
must start with 7 or 8 or 9
$3 \times 5 \times 4 \times 3$
$= 180$ possibilities

Ends in 8
must start with 7 or 9
$2 \times 5 \times 4 \times 3$
$= 120$ possibilities

So, a total of 660 even numbers which exceed 70 000

$$\text{P(even which exceeds 70 000)} = \frac{660}{2\,520} = 26{,}19\%$$

12. Fundi arranges her eight books randomly on her shelf. What is the probability that her three Maths books are all next to one another? (3)

total number of arrangements = 8! = 40 320

number of arrangements when the Maths books are together?
Put a rubber band round them
We now have 6 books which can be arranged in 6! ways
but for each of these there are 3! arrangements of the Maths books in the band
so, a total of 6! × 3!

$$P(\text{Maths books together}) = \frac{6! \times 3!}{8!}$$

$$P(\text{Maths books together}) = 10{,}71\%$$

13. Consider the letters

M A T H E M A T I C A L

a. How many different "words" can be formed using all the letters? (3)

If all letters were different it would be 11!
but we need to divide by 2! for the repeated Ms
and 2! for the repeated Ts
and 3! for the repeated As

so, a total of $\frac{11!}{2! \times 2! \times 3!} = 1\,663\,200$ words are possible

b. If such a word is chosen at random then what is the probability that it does NOT start and end with the same letter? (5)

Number which *DO* start and end with same letter:

starting and ending with M : $\frac{9!}{2! \times 3!} = 30\,240$

starting and ending with T : $\frac{9!}{2! \times 3!} = 30\,240$

starting and ending with A : $\frac{9!}{2! \times 2!} = 90\,720$

So, $P(\text{start and end with same letter}) = \frac{30\,240 + 30\,240 + 90\,720}{1\,663\,200}$

So, $P(\text{start and end with same letter}) = \frac{1}{11}$

So, $P(\text{DON'T start and end with same letter}) = \frac{10}{11}$

14. Alexandra has cards with the following digits on:

6	2	4	1	8

She has just one of each card and she is forming numbers using **some or all of** her cards.

For example:

21 OR 286 OR 1

a. How many different numbers can she form in this way? (4)

1 digit – 5 possibilities
2 digit – $5 \times 4 = 20$ possibilities
3 digit – $5 \times 4 \times 3 = 60$ possibilities
4 digit – $5 \times 4 \times 3 \times 2 = 120$ possibilities
5 digit – $5 \times 4 \times 3 \times 2 \times 1 = 120$ possibilities

So, a total of 325 possibilities

b. What is the probability that the number she forms is bigger than 5000? (4)

this would need to be a 4 digit number starting in 4, 6 or 8
There are $3 \times 4 \times 3 \times 2 = 48$

OR a 5 digit number of which there are 120

so, there are a total of 168 numbers which exceed 5000

$$P(\text{she forms a number bigger than } 5\,000) = \frac{168}{325}$$

$$P(\text{she forms a number bigger than } 5\,000) = 51{,}69\%$$

15. A group of 6 friends goes to the movie. Johan hopes he won't be sitting next to Pieter as they have recently had a disagreement. What is the probability that they are NOT seated next to one another? (4)

number of ways they ARE together?
Put a band around Johan and Pieter
so we have 5 'people' who can be arranged in $5! = 120$ ways
but for each of these there are $2! = 2$ arrangements of Johan and Pieter in the band
so, a total of 240 ways they ARE together

There are a total of $6! = 720$ arrangements of 6 people
so, there are 480 arrangements where they are *NOT* together

$$P(\text{not together}) = \frac{480}{720} = \frac{2}{3}$$

16. Nonhlanhla is investigating whether people's hot drink preference is dependent of their left or right handedness. She collects the following data:

	Prefer tea	Prefer coffee	TOTAL
Left handed	19	76	95
Right handed	213	852	1065
TOTAL	232	928	1160

a. Complete the table

b. Determine whether people's hot drink preference is dependent of their left or right handedness. Justify your answer with a calculation

$$P(\text{left handed} \cap \text{prefer tea}) = \frac{19}{1\,160} = 0{,}01637931$$

$$P(\text{left handed}) \times P(\text{prefer tea}) = \frac{95}{1\,160} \times \frac{232}{1\,160} = 0{,}01637931$$

so, hot drink preference is independent of left / right hand preference