Grade 12 Mathematics

Revision Workshop on Sequences, Functions and Trigonometry

28th February 2026

S·I·M·I

SHAYA IZIBALO MATHS INSTITUTE

NAME: ____

Paper 1 – Sequences and Series

	Term	**Sum**
Arithmetic	$T_n = a + (n-1)d$	$S_n = \frac{n}{2}[2a + (n-1)d]$ $\quad S_n = \frac{n}{2}[a + T_n]$
Geometric	$T_n = ar^{n-1}$	$S_n = \dfrac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \dfrac{a}{1-r}$; $-1 < r < 1$ and we have convergence
Quadratic	$T_n = an^2 + bn + c$ $a = \dfrac{\text{second difference}}{2}$. We can then work out b using the fact that $3a + b$ = first term of first differences Finally, we can work out c using the fact that $T_1 = a + b + c$	

Questions on Sequences and Series

1. Determine, showing working:

a. T_{20} of 2 ; 6 ; 10 ; 14 ; ...

b. T_{12} of 2048 ; 1 024 ; 512 ; 256 ; ....

c. S_{30} of 3 ; $4\frac{1}{2}$; 6 ; $7\frac{1}{2}$; ....

d. S_∞ *of* 9 ; 6 ; 4 ; ...

e. T_{10} of $2 ; 4 ; 8 ; 14 ; 22 ; ....$

f. $\sum_{i=3}^{15} (3i + 2)$

2. Determine k if $\sum_{i=4}^{23} (ki - 2) = 1\ 310$ showing all working details.

3. Which is the first term to exceed 10 000 000 in the sequence:

$10 ; 11 ; 12\frac{1}{10} ; 13\frac{31}{100} ; ....$

4. An arithmetic sequence has $T_9 = 13$ and $S_{10} = 60$. Find the first three terms.

5. A geometric sequence has $T_2 = 1\,800$ and $T_4 = 800$. Find two possible values for T_3

6. Which term in the sequence -1 ; 6 ; 17 ; 32 ; .... is equal to 899?

7. Find p if:

a. $2p + 3$; $4p + 2$ and $5p + 3$ are consecutive terms of an arithmetic sequence.

b. $3p + 1$; $2p + 4$ and $p + 8$ are consecutive terms of a geometric sequence.

c. $p - 1$; $2p - 1$; $4p - 2$ and $5p$ are consecutive terms of a quadratic progression.

8. A radioactive substance weighing 8 kg loses 9% of its mass each week. After how many weeks will it first weigh less than a kilogram?

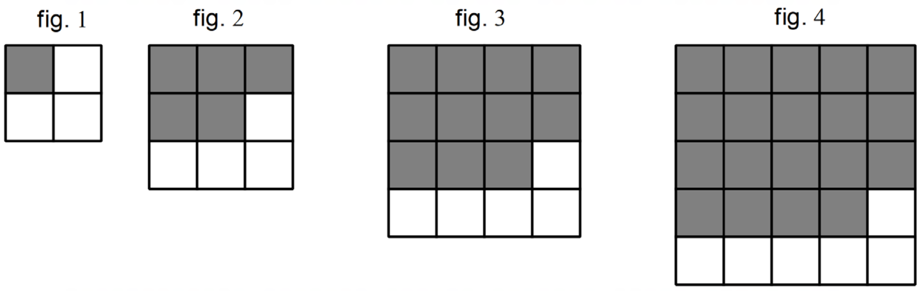
9. A tree grows 1,2 m in its first year. Each year thereafter it grows by 20% of the amount it grew the year before. What is the maximum height it will ever reach?

10. For what value(s) of k will $\sum_{i=1}^{\infty} (-3k+4)^i$ converge?

11. The first differences of a quadratic sequence are given by the formula $T_n = 2n + 2$. Find the 20th term of the quadratic sequence if the 10th term is 111.

12. Which term in the sequence 5 ; 12 ; 23 ; 38 ; 57 ; .... is equal to 597?

13. Which figure will have 1891 shaded squares?

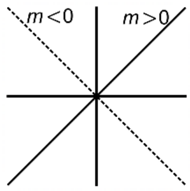


Paper 1 – Functions and Inverses

The Linear function – straight line

Standard form: $y = mx + c$

Increasing if $m > 0$
Deceasing if $m < 0$



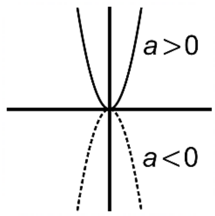
The Quadratic function – parabola

Parent function: $y = ax^2$

Standard form: $y = ax^2 + bx + c$

Axis of symmetry: $x = -\dfrac{b}{2a}$

“happy” if $a > 0$ and “sad” if $a < 0$

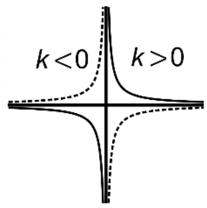


The hyperbolic function – hyperbola

Parent function: $y = \dfrac{k}{x}$

Axes of symmetry are $y = x$ and $y = -x$

Asymptotes are $y = 0$ and $x = 0$
Top right & bottom left if $k > 0$
Top let and bottom right if $k < 0$

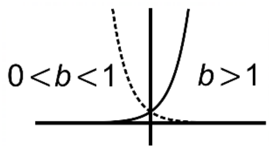


Standard form: $y = \dfrac{k}{x + p} + q$

The exponential function – exponential

Parent function: $y = ab^x$

Asymptote is $y = 0$
Decreasing if $0 < b < 1$
Increasing if $b > 1$

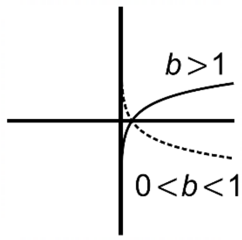


Standard form: $y = ab^x + c$

The logarithmic function – log

Parent function: $y = \log_b x$
Standard form: $y = \log_b(x + p) + q$

Asymptote is $x = 0$



Transformations

If, for example, $f(x) = x^2 + 2x + 6$ then:

$f(-x) = (-x)^2 + 2(-x) + 6$ — **reflects f in the y-axis**

$-f(x) = -(x^2 + 2x + 6)$ — **reflects f in the x-axis**

$f(x + p) = (x + p)^2 + 2(x + p) + 6$ — **shifts f left ($p > 0$) or right ($p < 0$)**

$f(x) + q = x^2 + 2x + 6 + q$ — **shifts f up ($q > 0$) or down ($q < 0$)**

$f^{-1}(x)$ — **reflects in the line $y = x$**

Questions on Functions and Inverses

1. For each of the following say, with justification, whether we have a function.

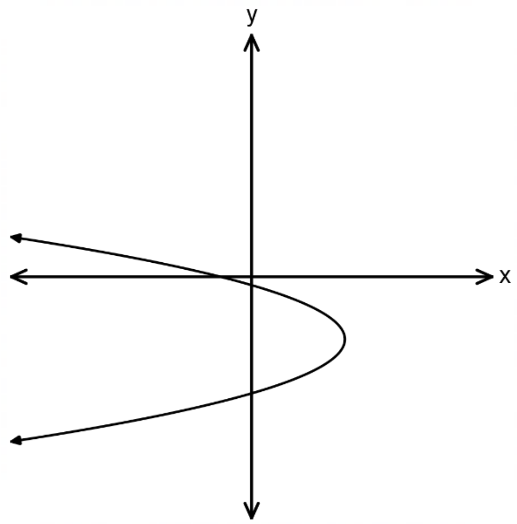
a. (-2;5) (1;3) (7;8) (1;5) (9;11)

b. $g(x) = \pm\sqrt{x+4}$

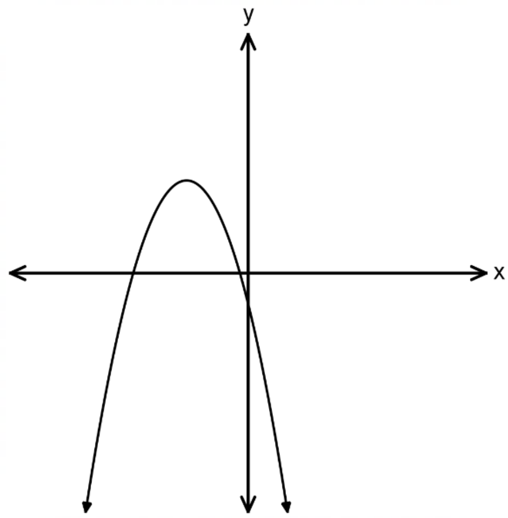
c. (-3;1) (2;5) (4;1) (5;6) (7;9)

d. $y = \sin x$

a.



f.



2. Give the domain and range of each of the following functions:

a. $y = \dfrac{12}{x-4} + 3$

b. $y = \log_2(x+4)$

a. $y = 0{,}5^x - 9$

d. $y = -(x+4)^2 + 9$

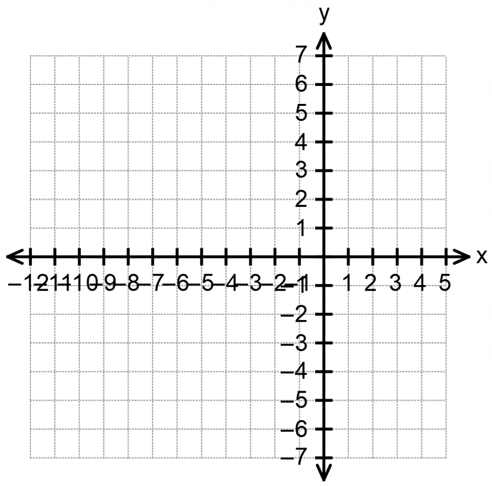
3. Find the turning point and the range of the following functions by completing the square:

a. $y = x^2 + 6x + 11$

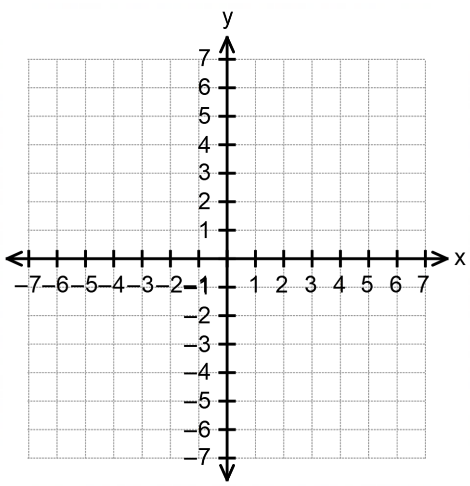
b. $y = -x^2 + 8x - 6$

4. Draw graphs of the following functions showing all points of interest. Where there are asymptotes, they should be drawn (with dotted lines) and labelled.

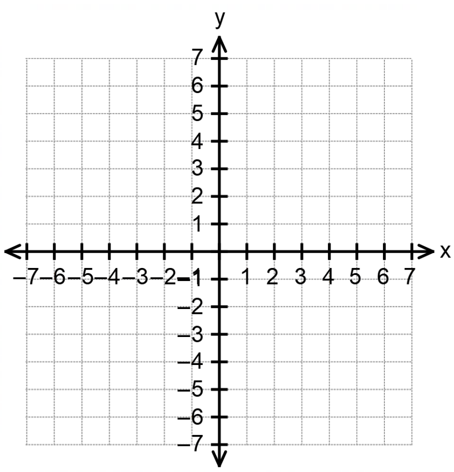
a. $y = \dfrac{-9}{x+4} - 2$



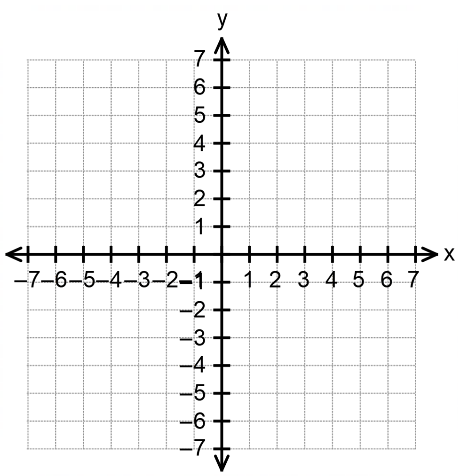
b. $y = 0{,}5^x - 4$



c. $y = \log_2(x + 4)$



d. $y = x^2 - 5x + 6$



5. Give, in standard form, the equation of the function which results when:

a. $y = x^2 + 3x - 5$
is shifted 3 left and 2 up

b. $y = 3^x + 5$
is reflected in the x-axis

b. $y = \dfrac{7}{x+3} - 2$
is reflected in the y-axis

d. $y = 3^x - 5$
is reflected in the line $y = x$

6. Find the inverses of the following functions in the form:

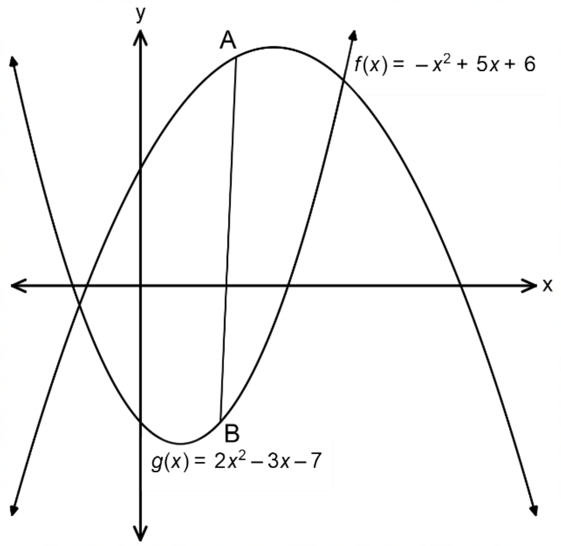
a. $f(x) = 3x - 5$

b. $f(x) = 2 \times 3^x + 6$

c. $f(x) = \log_2(x+3) - 5$

d. $f(x) = x^2 - 4$
give a domain
restriction which
will ensure that $f^{-1}(x)$
is a function

7. Consider the sketch below. AB is a vertical line with A on *g* and B on *f*.

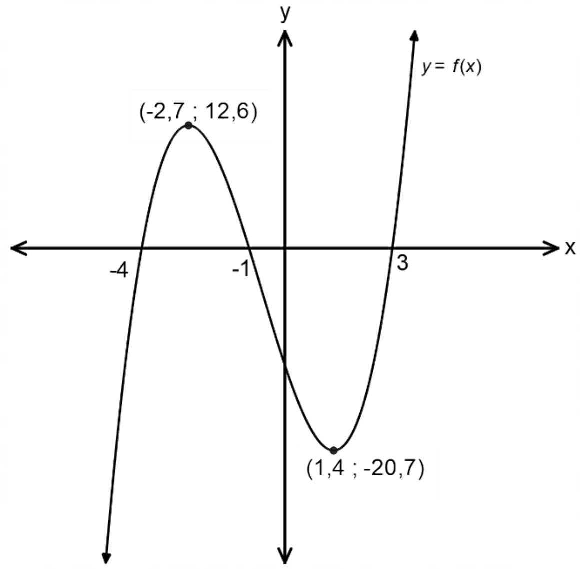


Find the maximum length of AB if it is on the interval where *g* exceeds *f*.

8. Find the equations of the axes of symmetry of $y = -\dfrac{12}{x+3} - 4$

9. For what value(s) of k will $y = x^2 + 3x - 3$ and $y = 2x + k$ not intersect one another?

10. Consider the diagram below:



a. For what value(s) of t will $f(x) + t = 0$ have exactly one root?

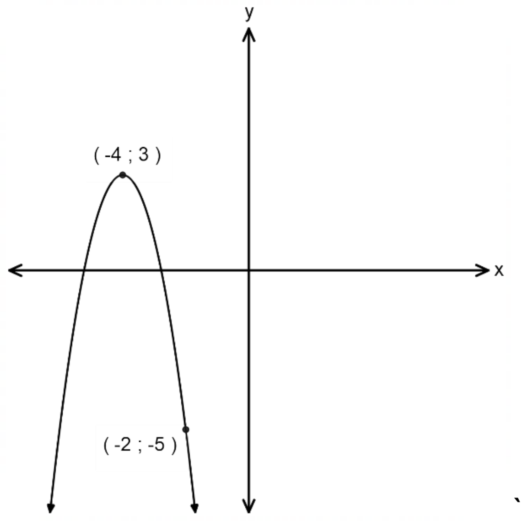
b. For what value(s) of k if will $f(x + k) = 0$ have only positive roots?

c. For what value(s) of t will $f(x) + t = 0$ have three distinct roots?

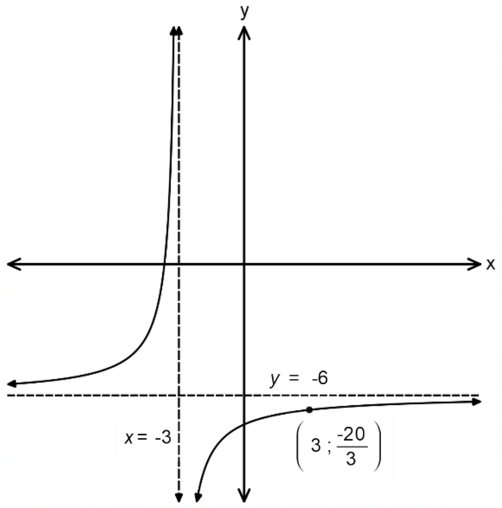
d. For what value(s) of k if will $f(x + k) = 0$ have 3 roots of the same sign?

11. Find the equations of the following functions:

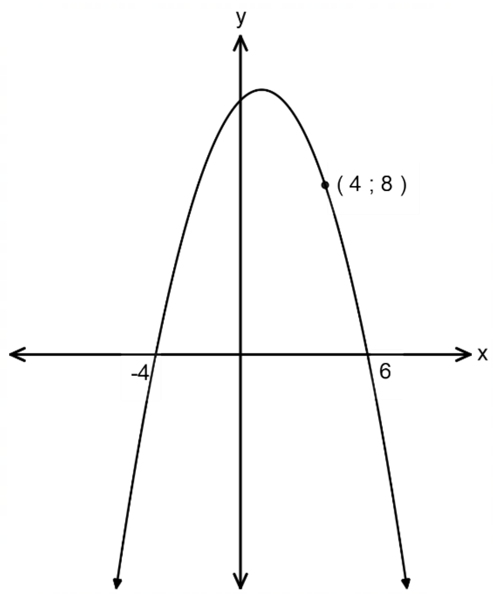
a.



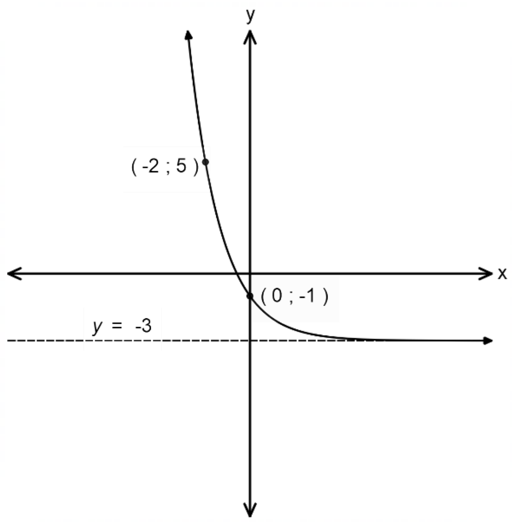
b.



c.



d.



PAPER 2 - Trigonometry

Not provided:

$\sin\theta = \frac{y}{r}$ or $\frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{x}{r}$ or $\frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{y}{x}$ or $\frac{\text{opp}}{\text{adj}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $\cos^2\theta = 1 - \sin^2\theta$ $\sin^2\theta = 1 - \cos^2\theta$

Provided:

In Δ ABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2}$ ab.sinC

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$ $\sin 2\alpha = 2\sin\alpha\cos\alpha$

1. Simplify without a calculator:

$\cos 15°$

$\cos 195°\sin 285° + \cos 105°\sin 15°$

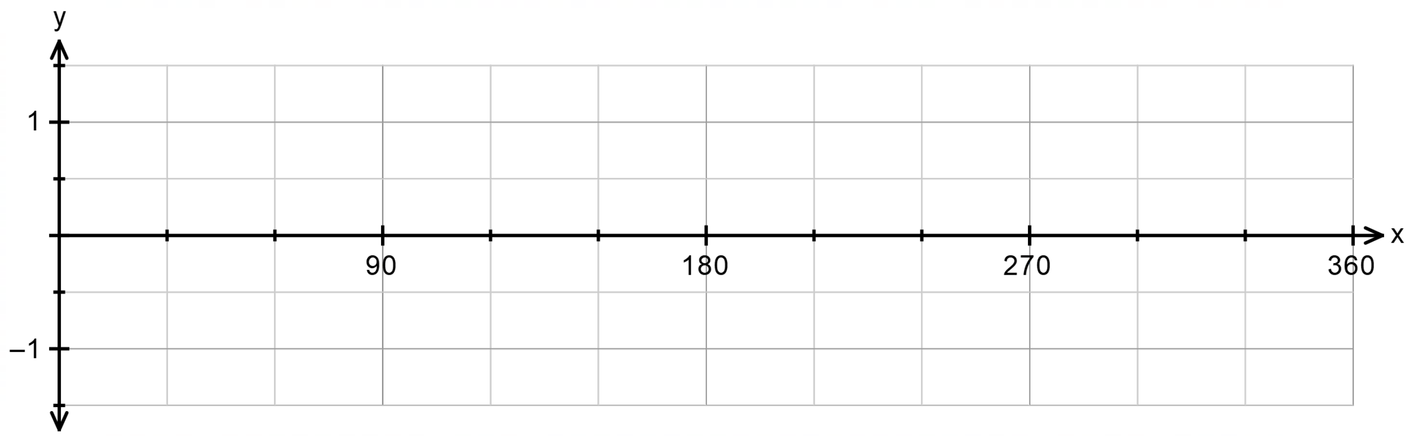
$$\frac{\sin 140°\cos 240°}{\sin 340°\sin 110°\cos 180°}$$

2. If $\cos 13° = m$ then give the following in terms of m:

$\cos 193°$ $\sin 283°$ $\sin(-347°)$

$\cos 386°$ $\cos 6,5°$

3. Draw on the same set of axes:
$f(x) = \sin x \cos 30° + \cos x \cos 30°$ and $g(x) = \cos^2 x - \sin^2 x$



Calculate the points of intersection of the two graphs:

4. Solve $\cos 2\theta \cos 10° + \sin 2\theta \sin 10° = -\frac{\sqrt{3}}{2}$ for $\theta \in [0°;360°]$

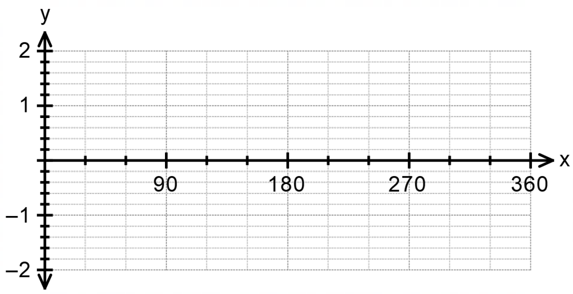
5. Prove:

a. $\frac{\sin 3x}{\sin x} + \frac{\cos 3x}{\cos x} = 4\cos 2x$

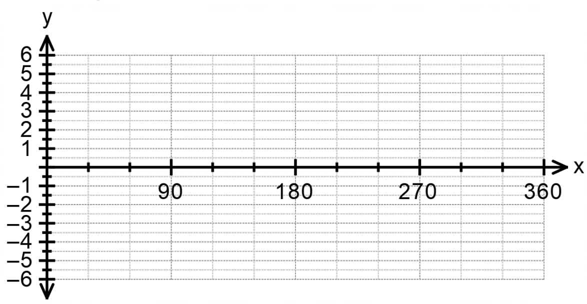
b. $\frac{2\sin^2\theta}{1+\cos 2\theta} = \tan^2\theta$

6. Sketch the following graphs on the axes provided:

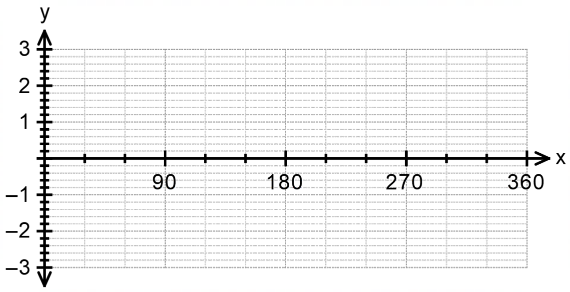
a. $y = \sin 2\theta - 1$



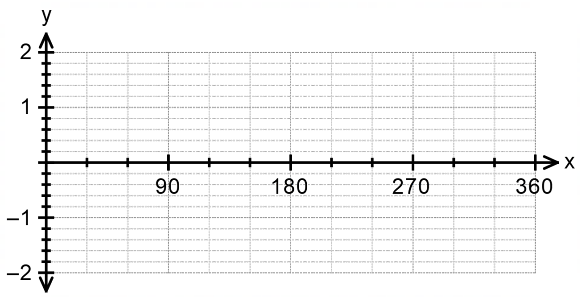
b. $y = -2\tan\theta$



c. $y = 2\cos\theta + 1$

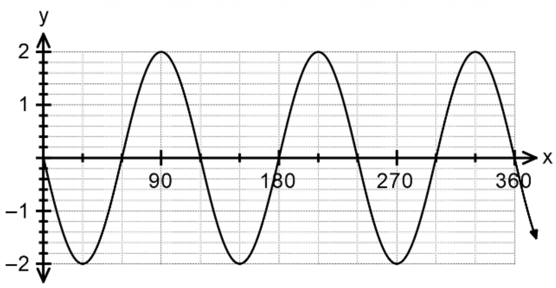


d. $y = \sin\theta\cos 30° - \cos\theta\sin 30°$

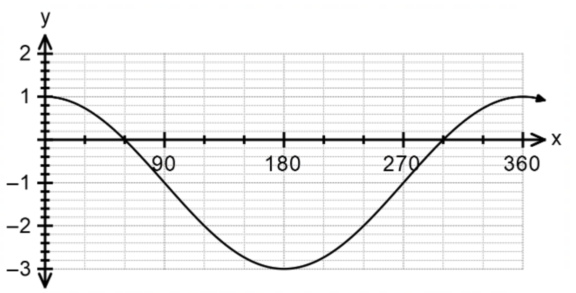


7. Find the equations of the following functions:

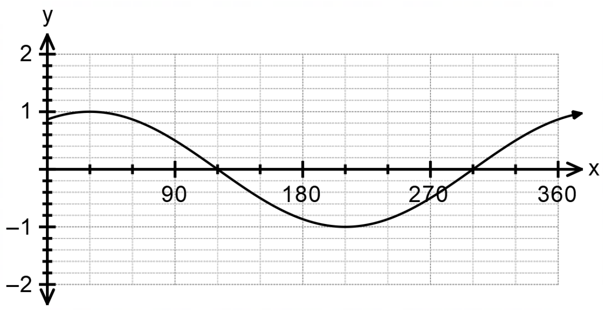
a.



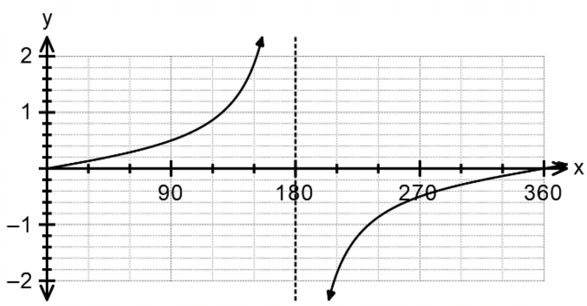
b.



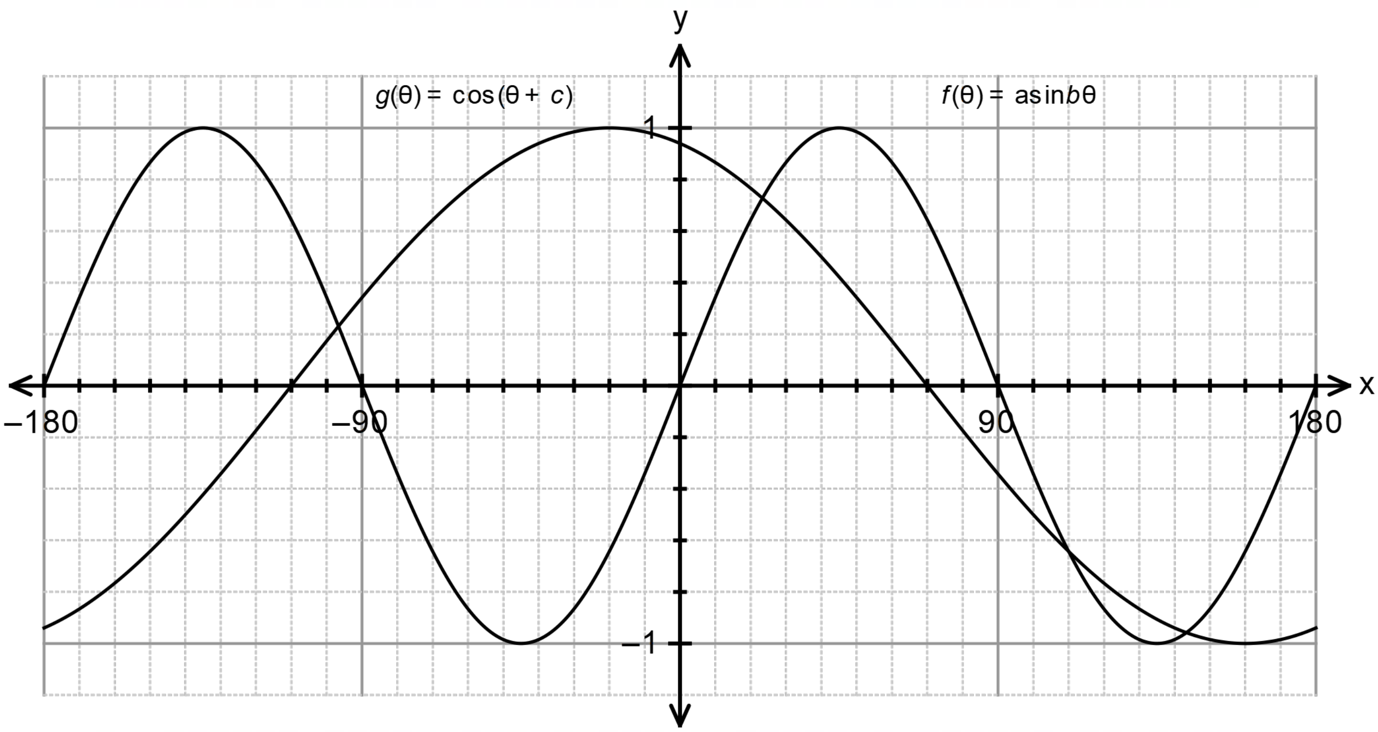
c.



d.



8. Consider the following functions drawn below for $\theta \in [-180°;180°]$



a. Determine the values of a, b and c.

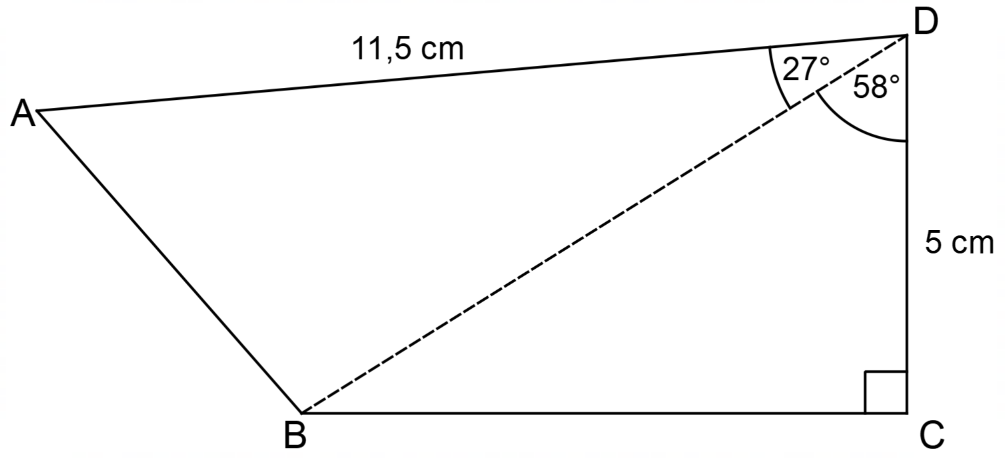
b. What is the period of f?

c. What is the amplitude of f?

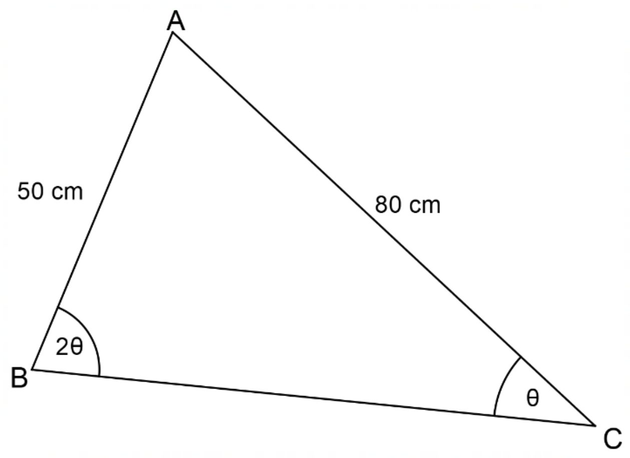
d. Determine the x-values of the points of intersection on the given domain.

e. Give the value(s) of θ on the given domain for which $g(\theta) > f(\theta)$

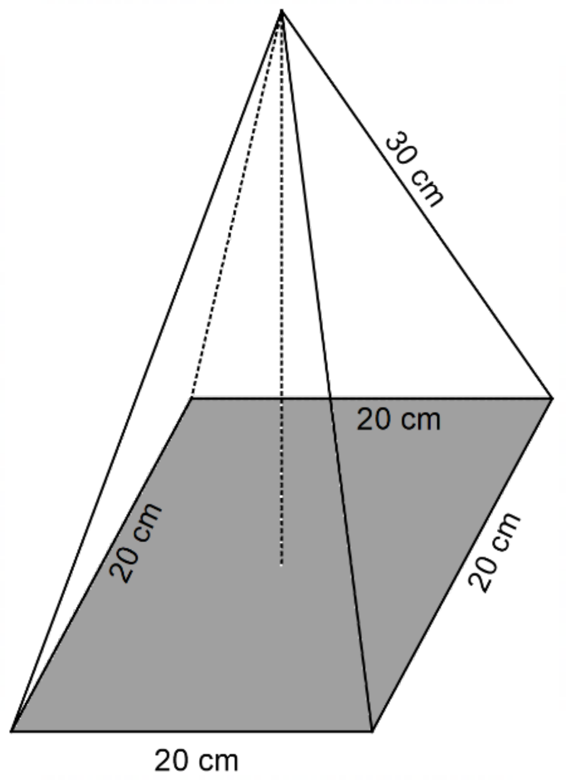
9. Find the perimeter of the shape below. Remember: don't include the dashed line!



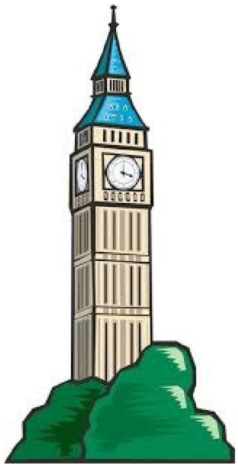
5. Find the area of ΔABC



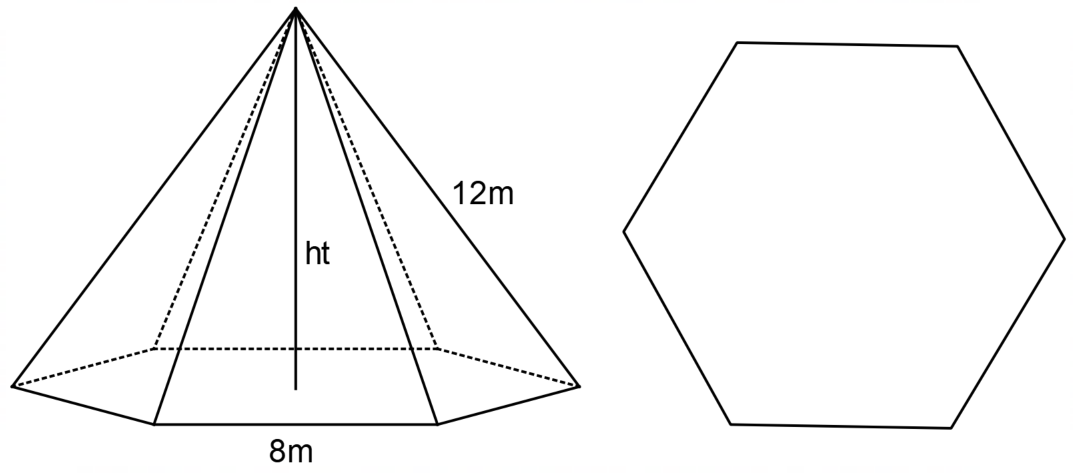
10. Consider the pyramid below. It has a square base with sides of 20 cm. If the slant height (s) is 12 cm, then how high is the pyramid?



9. Standing at a point the angle of elevation of a tower is 40°. The foot of the tower is in the same horizontal plane as the flat ground on which I am standing. When I walk 20m closer the angle of elevation increases to 60°. How high is the tower? Hint: let the height of the tower be *h* and my initial distance from the foot of the tower be *d*.



10. A tent has a hexagonal base with sides of 8m. The slant sides are 12m.



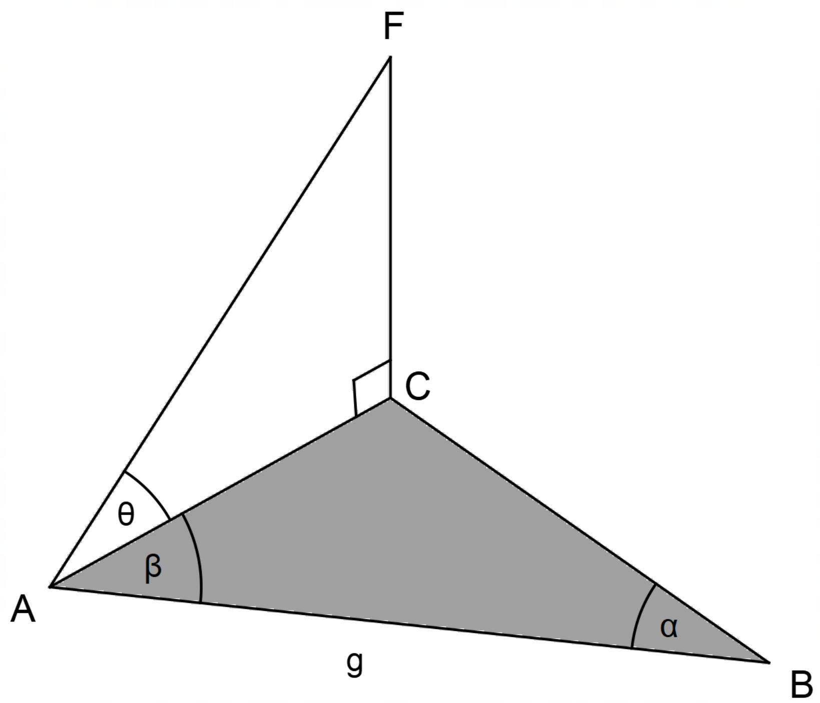
a. How high is the tent?

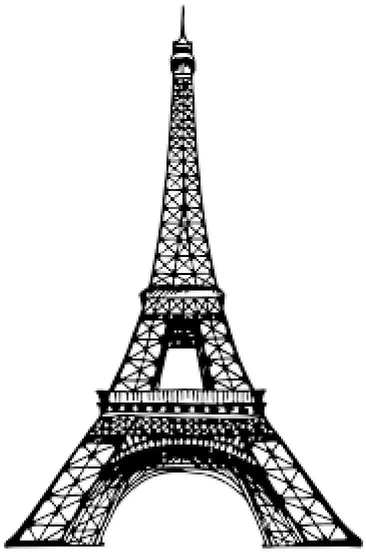
b. What area of canvas will be needed to make the sides of the tent?

c. What is the volume of the tent. Remember that volume of a right pyramid is given by:

$$\text{Vol} = \frac{1}{3} \times \text{area of base} \times \perp \text{height}$$

11. In the picture A, B and C are in the same horizontal plane. C is the foot of a **vertical** tower, FC. The distance between A and B is *g*. The angle of elevation of F from A is θ.
$C\hat{A}B = \beta$ and $C\hat{B}A = \alpha$.





Show that $FC = \dfrac{g\sin\alpha\tan\theta}{\sin(\alpha+\beta)}$