Grade 11 Mathematics

Paper 1 Workshop on Exponents, Equations and Inequalities

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EXPONENTS

LAWS

1. $a^p \times a^q = a^{p+q}$

2. $a^r \div a^s = a^{r-s}$

3. $(a^r)^t = a^{rt}$

4. $(ab)^m = a^m b^m$

5. $\left(\frac{a}{b}\right)^k = \frac{a^k}{b^k}$

EXERCISE 1

Simplify, expressing your answers with positive exponents.

1. $\left(x^{\frac{1}{2}}\right)\left(x^{\frac{3}{4}}\right)$

2. $\left(x^{\frac{1}{2}}\right)^{\frac{3}{4}}$

3. $\left(x^{\frac{1}{2}}\right)^{-2}$

4. $\dfrac{24a^5b^{-3}}{6ab^{-2}}$

5. $\dfrac{2a^3b^{-1}}{a^{-3}} \times \dfrac{3a^2b^2}{(a^2)^3}$

6. $\left(\dfrac{a^2b^{-3}}{c^{-2}}\right)^{-1}$

7. $\dfrac{a^3b^{-2}}{a^2b} \div \dfrac{a^3b}{b^{-1}}$

8. $\left(\dfrac{a^3b^{-2}}{c^{-2}}\right)^2 \div \left(\dfrac{a^{-2}b^3}{c^3}\right)^{-2}$

EXERCISE 2

Simplify, expressing your answers with positive exponents.

1. $\dfrac{18^{x}(2.3^{1-x})^{2}}{2^{x-1}}$

2. $\dfrac{12^{2x-1}.9^{x+2}}{4^{x}.18^{2x}}$

3. $\dfrac{4^{n+2} \times 36^{-n-1}}{45^{-n+1} \times 5^{n-1} \times 81^{-1}}$

4. $\dfrac{12^{2x+1} \times 9^{x-2}}{36^{2x-1}}$

EXERCISE 3

Simplify, expressing your answers with positive exponents.

1. $\dfrac{3.2^{m}-4.2^{m+2}}{2^{m}-2^{m+1}}$

2. $\dfrac{2^{2x}-2^{x}-6}{2^{x}+2}$

3. $\dfrac{3^{2n+1}-9^{n}}{6\times 3^{2n-1}}$

4. $\dfrac{6^{n}+3^{n+1}}{2^{n}+3}$

ROOTS AND SURDS

We will make extensive use of this definition:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \qquad (n \geq 2 \text{ and } m \in \mathbb{Z})$$

The following surd laws hold. They can each be shown to be true by changing to exponents and using our exponent laws. Each of them applies from left to right or from right to left. Space has been left for your teacher to provide you with an example or two.

1. $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ "the product of some roots is equal to the root of the product"
   Note: the root type **must** be the same

2. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

3. $\left(\sqrt[m]{a}\right)^n = \sqrt[m]{a^n}$

4. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$ "the quotient of two roots is equal to the root of the quotient"

   Note: the root type **must** be the same

EXERCISE 4

Simplify without a calculator

1. $2\sqrt{8} - 4\sqrt{32} + 3\sqrt{50}$

2. $\sqrt{50}\left(\sqrt{18} + \sqrt{32}\right)$

3. $\left(2 - \sqrt{3}\right)^2 + \left(2 + \sqrt{3}\right)^2$

4. $\dfrac{\sqrt{x} - \dfrac{x}{2\sqrt{x}}}{3x^{\frac{3}{2}}}$

5. $\frac{3}{\sqrt{5}}$

6. $\frac{3}{\sqrt{5}+2}$

EXERCISE 5

Solve the following equations:

1. $2x^{\frac{2}{3}} = 32$

2. $x^{-\frac{3}{2}} = 64$

3. $x^{\frac{2}{3}} = 9$

4. $3x^{\frac{5}{7}} + 2 = 90$

EXERCISE 6

Solve the following equations:

1. $3^{1-2x} = 27$

2. $\left(\frac{1}{2}\right)^{x+1} = 2^{2x-4}$

3. $2^x + 2^{x+1} = 24$

4. $3.5^x + 5^{x+1} = 40$

5. $3^{2x} - 4.3^{x} + 3 = 0$

6. $4^{x} + 16 = 10.2^{x}$

7. $3^{2x+1} - 4.3^{x} + 1 = 0$

8. $3^{x} - 4 = \frac{45}{3^{x}}$

EQUATIONS AND INEQUALITIES

EXERCISE 1

Solve the following equations:

a) $x^2 - 2x = 80$

b) $x(x-2) = 24$

c) $2x^2 + 7x + 6 = 0$

d) $(2x+3) + 5x(2x+3) = 0$

e) $x(x-1) = 4(3x-10)$

f) $u(u+4) = 21$

EXERCISE 2

a) If $x = -2$ is a solution of the equation $4x^2 + px = -2$ where $p \neq 0$ then determine the value of p and, hence, the other solution.

b) If $x = 2$ is a root of the equation $px^2 - x + 5 - 3p = 0$, where $p \neq 0$ then determine the value of p and, hence, the other root.

EXERCISE 3

a) Create a quadratic equation which has roots 3 and -2
.

b) Create a quadratic equation which has roots 3 and -1 and a coefficient of x^2 equal to 3.

EXERCISE 4

Solve for x:

1) $\frac{x}{x-2}=\frac{1}{x-3}+\frac{2}{x-2}$

2) $\frac{x}{x-2}=\frac{1}{x-3}-\frac{2}{2-x}$

3) $\frac{1-10x}{4x^2-9}=\frac{4}{3-2x}+\frac{2x-1}{2x+3}$

EXERCISE 5

1) Solve for x: $(x^2 - x)^2 = 8(x^2 - x) - 12$

2) $x^2 - x - 8 + \frac{12}{x^2 - x} = 0$

3) $(x^2 - x)^2 + 12 = 8(x^2 - x)$

4) $(p^2 - 2p)^2 - 2(p^2 - 2p) = 3$

EXERCISE 6

Solve for x:

1) $\sqrt{6+x}-x=0$

2) $4\sqrt{x-2}-1=x$

3) $\sqrt{2x+1}-x=-1$

EXERCISE 7

Solve the following equations:

a) $(x-4)^2-12=0$

b) $(x+2)^2=16$

c) $3(x-2)^2=12$

EXERCISE 8

Solve the following equations by completing the square:

a) $x^2 + 6x - 2 = 0$

b) $x^2 - 8x - 1 = 0$

c) $2x^2 - 4x - 5 = 0$

d) $3x^2 - 5x - 2 = 0$

e) $ax^2 + bx + c = 0$

EXERCISE 9

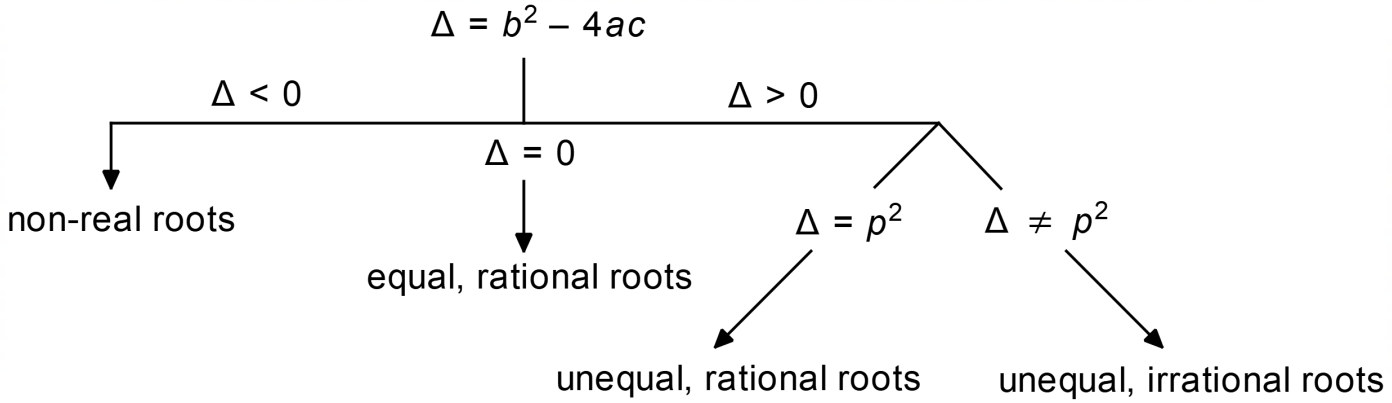
Solve the following equations using the quadratic formula. Give your answers to 2 decimal places:

a) $x^2 + 5x - 7 = 0$

b) $x^2 + 6x + 4 = 0$

c) $2x^2 = 3x + 1$

d) $x(x + 3) = 9$



EXERCISE 10

Without solving determine the nature of the roots of the following equations:

a) $10x^2 - x - 2 = 0$

b) $2x - \frac{4}{x} = 8$

c) $x(x+2) = 8$

d) $x^2 + 5x + 7 = -x - 2$

EXERCISE 11

a) For what value(s) of m will the roots of $2mx^2 - (m-2)x + m + 1 = 0$ with $m \neq 0$ be:

(i) Equal?

(ii) Real?

b) For which value(s) of k will the equation $x^2 - 11 = 2k(x-3) - 2x$ have non-real roots?

EXERCISE 12

Solve simultaneously:

a) $y = 2x^2 + 6x - 8$
$y - x = 4$

b) $2x + 3y = 1$ and $2x^2 + xy - 6x + 3y - y^2 = 0$

EXERCISE 13

Solve the following inequalities. Give your answer in inequality and interval notation and show them on the number lines provided:

a) $-x^2 + 5x \geq 6$

b) $x^2 + 2x - 3 > -x + 1$

c) $(x+2)(x-5)>8$

d) $(x-2)(x+3)\leq 6$