Grade 10 Mathematics

Paper 1 Revision Workshop on Algebraic expressions, Exponents, Equations and Inequalities

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PRODUCTS AND FACTORS

Consider the following:

$2(x+3)$

$2(a+b-3)$

$(a+b)(c+d)$

$(a+b)(c+d+e)$

The summary is that: everything in every bracket must multiply by everything in the other bracket. If we have three brackets we must multiply any two of them and then multiply in the third bracket.

Expand and simplify:

$(a+b)(c+d)(e+f)$

$(x+3)^2$

$(x+3)^3$

$2(x+3)^3$

Consider the following:

$11-9$ vs $11-(5+4)$ vs $11-5+4$

Now simplify the following:

a) $3(x-4)-6(x-1)$

b) $(x+3)(x-5)-(x-1)(x-6)$

c) $(x+4)^2-(x-3)^2$

d) $(x^2+3x-5)(x^2-4x-1)$

e) $3(x-2)(x+4)-2(x+1)^2$

f) $(x+2)^3-(x+1)^3$

Factorising

To factorise an expression means to write it as a product of its factors. Sometimes more than one step is required. We must always check that it is both **CORRECT** (it multiplies back) and **COMPLETE** (none of the factors can factorise further).

STEP 1 The first thing we do is look for a highest common factor:

a) $3x-6$

b) $4x^3yp-8x^2y^3p$

c) $12x^3y^2-4xy^2+8x^4y^3$

Now consider these ones which have **four or more terms**. While there may not be a common factor in all the terms, there may be some pairs (or triples) which have a common factor:

a) $3a+3b+ac+bc$

b) $de+df+fg+eg$

c) $2a-4+ac-2c+2d-ad$

Note: any subtraction can be switched by changing the sign in front of it – a switcheroo!

$a-b=-(b-a)=-b+a$

Now try these:

a) $3a-3b+db-da$

b) $(2-x)(a+b)+(x-2)(c+d)$

STEP 2 If we have 2 terms we could have any of the following:

Difference Of Two Squares $a^2 - b^2 = (a - b)(a + b)$

Difference Of Two Cubes $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Sum Of Two Cubes $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

Now try factoring these. Remember to look for a common factor first:

a) $a^2 - 100$ b) $x^3 + 27$ c) $3x^3 - 24$

d) $2x^4 - 32$ e) $5x^3 + 5$ f) $(3x + 5y)^2 - (2x + y)^2$

STEP 3 If it has 3 terms it could be a trinomial

If the second sign is positive then both signs will be the same and the same as the first sign.
If the second sign is negative then one sign will be positive and the other negative

We want “factors of back number that will ‘second sign’ to give middle term”

Try these. Remember to check for a common factor first:

a) x^2+5x+6 b) $2x^2-2x-12$ c) $x^2-2x-24$

d) $3x^2-3x-90$ e) $a^2+9a+14$ f) $gp^2-11gp+24p$

g) $2x^2+7x+6$ h) $6x^2-19x+10$ i) $12x^2-5x-2$

Now for some mixed factorising practice. Factorise fully:

a) $3x^2y-5xy$ b) $2x^2-18$ c) $x^2-9+3a-ax$

d) $3x^3+81$ e) $a^2+a-6+2b-ab$ f) $5x^4-5$

Simplifying algebraic fractions

Consider simplifying the fraction:

$$\frac{4}{12}$$

We cannot add / subtract from the numerator (top) and denominator (bottom). We can only multiply or divide both by the same number.

Now consider simplifying the fraction:

$$\frac{x^2-4}{2x-4}$$

We must always factorise before dividing. Don't use the word cancel – it's problematic!

Simplify the following fractions:

a) $\frac{3x-6}{4-2x}$ b) $\frac{x^2-1}{x^3-1}$ c) $\frac{x^2-2x-8}{x^2-4}$

Multiplying fractions. We simply multiply the tops and multiply the bottoms. Mixed numbers must first be converted to improper fractions.

e.g. $\frac{2}{3}\times\frac{5}{7}$ $\frac{3}{4}\times\frac{2}{5}$ $2\tfrac{1}{2}\times\frac{1}{9}$

Sometimes it is easier to simplify first. Consider:

$\frac{5}{18}\times\frac{9}{55}$ $\frac{x-2}{x+3}\times\frac{x^2+6x+9}{x^2-4}$

Simplify the following:

a) $\dfrac{3x+6}{x^3+8} \times \dfrac{2x^2-4x+8}{3p}$

b) $\dfrac{3a-3b+cb-ac}{a^2-b^2}$

Dividing fractions. We simply tip the second fraction upside down and then multiply them.

$50 \div \dfrac{1}{2}$

$\dfrac{3}{x} \div \dfrac{5y}{2x}$

Notice, dividing in half and dividing by half are different!

a) $\dfrac{x^2-4}{3y} \div \dfrac{3x+6}{9y}$

b) $\dfrac{\dfrac{4x^3y}{2x^2-2}}{\dfrac{8x^4y^3}{5x-1}}$

Adding and subtracting fractions. We must first get them to the same type (denominator)

Consider adding:

2$ + R5

30 cm +20 mm

$\frac{1}{3} + \frac{5}{12}$

$\frac{2}{3}+\frac{1}{7}-\frac{1}{2}$

$\frac{1}{x}+\frac{3}{2y}+\frac{7}{y}$

Simplify, remembering order of operations (BODMAS)

a) $\frac{1}{x+2}+\frac{3}{x-2}$

b) $\frac{5}{x^2-4}+\frac{3}{2-x}+\frac{6}{2+x}$

c) $\frac{(2x+3)(x-2)-(x+1)(2x+3)}{6x+9}$

d) $\frac{3}{xy}+\frac{2}{5x}\times\frac{3}{4y}$

e) $\frac{1}{a^2-b^2}-\frac{3}{b-a}+\frac{5}{2a+2b}$

EXPONENTS

LAWS

1. $a^p \times a^q = a^{p+q}$

2. $a^r \div a^s = a^{r-s}$

Two special consequences!

3. $(a^r)^t = a^{rt}$

4. $(ab)^m = a^m b^m$

5. $\left(\frac{a}{b}\right)^k = \frac{a^k}{b^k}$

Simplify, expressing your answers with positive exponents.

1. $\left(x^{\frac{1}{2}}\right)\left(x^{\frac{3}{4}}\right)$

2. $\left(x^{\frac{1}{2}}\right)^{\frac{3}{4}}$

3. $\dfrac{24a^5b^{-3}}{6ab^{-2}}$

4. $\dfrac{2a^3b^{-1}}{a^{-3}} \times \dfrac{3a^2b^2}{(a^2)^3}$

5. $\left(\dfrac{a^2b^{-3}}{c^{-2}}\right)^{-1}$

6. $\dfrac{a^3b^{-2}}{a^2b} \div \dfrac{a^3b}{b^{-1}}$

EXERCISE 2

Simplify, expressing your answers with positive exponents. In these ones we need to prime factor our powers to get common bases – "it's all about that base!"

1. $\dfrac{18^x(2.3^{1-x})^2}{2^{x-1}}$

2. $\dfrac{12^{2x-1}.9^{x+2}}{4^x.18^{2x}}$

3. $\dfrac{4^{n+2} \times 36^{-n-1}}{45^{-n+1} \times 5^{n-1} \times 81^{-1}}$

4. $\dfrac{12^{2x+1} \times 9^{x-2}}{36^{2x-1}}$

EXERCISE 3

Simplify, expressing your answers with positive exponents. Remember to factorise first!

1. $\dfrac{3^{m+2}-3^{m}}{2\cdot 3^{m}}$

2. $\dfrac{3.2^{m}-4.2^{m+2}}{2^{m}-2^{m+1}}$

3. $\dfrac{4\times 5^{m+1}-5^{m-1}}{3\times 5^{m}}$

4. $\dfrac{3^{2n+1}-9^{n}}{6\times 3^{2n-1}}$

SOLVING EQUATIONS AND INEQUALITIES

Two important ideas when solving equations and inequalities:

Getting dressed and undressed

A scale metaphor

Solve for *x*:

a) $2x + 3 = 11$

b) $2(x - 4) + 1 = 19$

c) $\frac{x}{2} + \frac{x}{3} = 10$

d) $ax - b = c$

e) $4x + 3 = 7x + 19$

f) $ax + b = cx + d$

g) $5x^{\frac{7}{9}} - 1 = 299$

h) $\frac{x}{a} + c = \frac{x}{d} + b$

i) $3(x + 2) - 1 - x = 2x + 3$

j) $\frac{x - 2m}{a} = b$

k) $2x^2 + 3 = 78$

l) $\frac{\sqrt{x + 2}}{5} + 2 = 7$

m) $\dfrac{\sqrt{ax+b}}{c} = d + e$

Exponential equations are when the variable is in the exponent.

The strategy is to get one factor on each side with the same base. Then we can equate the exponents.

Solve for *x*:

a) $2^x = 8$

b) $5^{x-2} = 1$

c) $3^x \times 3 = 27^x$

d) $4^{x+2} \times 8^{x-3}$

e) $3 \times 4^{x+1} = 96$

$27^x = \left(\dfrac{1}{3}\right)^{x+5}$

Inequalities – similar to equations but for one important rule!

Interval notation – remember [or] means including the end points while (or) means excluding the end points.

Number line – remember a shaded dot means including the end point while an open dot means excluding the end point.

Solve, also giving your answers in interval notation and on a number line

a) $2x+3<11$

b) $\frac{3x+4}{4}-1\leq x$

c) $-2x+4\geq 12$

d) $-3<2x+7\leq 19$

e) $-11\leq\frac{-2x+3}{2}+1<19$

f) $-3<\frac{2x}{-5}+7<20$

QUADRATIC EQUATIONS. We have a term in x^2

We get 0 on right-hand side and factor the left-hand side.

If we can get $A.B = 0$ then $A = 0$ or $B = 0$

Solve for x:

a) $x^2 - x = 6$

b) $(x-3)(x+2) = 6$

c) $(x+2)^2 - 3(x+2) = 0$

d) $x^2 - 2x = 8$

e) $x + 1 = \frac{90}{x}$

f) $2x + 2 = \frac{60}{x}$

If we have **fractions** we must remember to check that our solutions do not make the original question undefined (zero underneath!). If so, we must cross them out.

g) $1 + \frac{1}{x-3} = \frac{x-2}{x-3}$

h) $\frac{x}{x-2} = \frac{1}{x-3} - \frac{2}{2-x}$

Sometimes it can help to make a **substitution**:

a) $(x^2 - x)^2 = 8(x^2 - x) - 12$

b) $x^2 - x - 8 + \dfrac{12}{x^2 - x} = 0$

If we have a **square root** then we must first isolate it before squaring both sides (properly! 😊)
However, we must also remember to check our solutions!

Consider $x + 3 = 5$

Solve:

a) $\sqrt{x+6} - x = 0$

b) $\sqrt{x+2} - 1 = x - 5$

c) $2\sqrt{x+3} - x = 3$

d) $\sqrt{x-5} + 2 = x - 5$

Simultaneous equations

Two methods – substitution and elimination

a) $y - 1 = 2x$ and $5x - 2y = 3$

b) $4x + 3y = 6$ and $-3x + y = -11$

c) $3x - 5y = -1$ and $7x + 5y = 31$

Word problems

a) 3 packets of chips and 4 drinks cost R66. 5 packets of chips and 2 drinks cost R68.

What does a packet of chips cost? What about a drink?

b) The length of a rectangle is 4m longer than the width. If the area is 77 m^2 then how wide is the rectangle?

c) Skulule buys some goldfish for R600. The bad news is that a cold spell kills them all. However, the good news is that when he goes to buy some more the price has dropped by R5 per fish with the result that he can get 50% more fish for the same amount of money. How much did they cost each initially?