Grade 12 Mathematics

Revision Workshop on Sequences, Functions and Trigonometry

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S·I·M·I

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NAME: ____

Paper 1 – Sequences and Series

2; 5; 8; 11;

	Term	Sum
Arithmetic	$T_n = a + (n-1)d$	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_n = \frac{n}{2}[a + T_n]$
Geometric	$T_n = ar^{n-1}$ 2; 6; 18;	$S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$ and we have convergence
Quadratic	$T_n = an^2 + bn + c$ $a = \frac{\text{second difference}}{2}$. * We can then work out b using the fact that $3a + b$ = first term of first differences * Finally, we can work out c using the fact that $T_1 = a + b + c$ *	

Questions on Sequences and Series

1. Determine, showing working:

a. T_{20} of 2 ; 6 ; 10 ; 14 ; ...

$T_n = a + (n-1)d$

$T_{20} = 2 + (19)4$

$= 78$

b. T_{12} of 2048 ; 1 024 ; 512 ; 256 ; ....

$r = \frac{T_4}{T_3}$

$T_n = ar^{n-1}$

$T_{12} = 2048\left(\frac{1}{2}\right)^{11}$

$= 1$

c. S_{30} of 3 ; $4\frac{1}{2}$; 6 ; $7\frac{1}{2}$; ....

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{30} = \frac{30}{2}[2(3) + 29(1{,}5)]$

$= 742{,}5$

d. S_∞ of 9 ; 6 ; 4 ; ...

$S_\infty = \frac{a}{1-r}$

$= \frac{9}{1-\frac{2}{3}}$

$= 27$

e. T_{10} of 2 ; 4 ; 8 ; 14 ; 22 ; ....

2 ; 4 ; 6 ; 8 .

2 ; 2 ; 2

$a = 1$

$3(1) + b = 2$

$b = -1$

$1 + (-1) + c = 2$

$c = 2$

$T_n = n^2 - n + 2$

$T_{10} = 92$

f. $\sum_{i=3}^{15}(3i + 2)$ = 11 + 14 + 17

SUM

15 − 3 + 1

$n = ?$

13 ✓

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{13} = \frac{13}{2}[22 + 12(3)]$

$= 377$

$n = 23 - 4 + 1$

$= 20$

2. Determine k if $\sum_{i=4}^{23}(ki - 2) = 1\,310$ showing all working details.

C

$(4k - 2) + (5k - 2) + (6k - 2)$

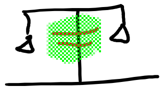
$d = k$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$1310 = \frac{20}{2}[2(4k - 2) + 19k]$

$=$

$d = k$



$1310 = \frac{20}{2}\left[2(4k-2) + 19k\right]$

$1310 = 10\left[27k - 4\right]$

$131 = 27k - 4$

$135 = 27k$

$5 = k$

3. Which is the first term to exceed 10 000 000 in the sequence:

10 ; 11 ; $12\frac{1}{10}$; $13\frac{31}{100}$; ....

Geometric $\quad r = \frac{11}{10} = 1,1$

$T_n = ar^{n-1}$

$10\ 000\ 000 = 10 \times 1,1^{n-1}$

$1\ 000\ 000 = 1,1^{n-1}$

$n - 1 = \log_{1,1} 1\ 000\ 000$

$n = 146$

4. An arithmetic sequence has $T_9 = 13$ and $S_{10} = 60$. Find the first three terms.

$a \quad d$

① $13 = a + 8d$

② $60 = 5\left[2a + 9d\right]$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$a = 13 - 8d$

$60 = 5\left[2(13 - 8d) + 9d\right]$

$12 = \left[26 - 16d + 9d\right]$

$-14 = -7d$

$2 = d \qquad a = 13 - 8(2) = -3$

$\boxed{-3; -1; 1}$

5. A geometric sequence has $T_2 = 1\,800$ and $T_4 = 800$. Find two possible values for T_3

$$① \quad 800 = ar^3$$
$$② \quad 1800 = ar$$
$$① \div ② \quad \frac{4}{9} = r^2$$
$$r = \pm \frac{2}{3}$$
$$T_3 = 1200 \quad \text{or} \quad -1200$$

6. Which term in the sequence -1 ; 6 ; 17 ; 32 ; .... is equal to 899?

7 ; 11 ; 15

4 ; 4

$$a = 2$$
$$3(2) + b = 7 \qquad b = 1$$
$$2 + 1 + c = -1 \qquad c = -4$$
$$T_n = 2n^2 + n - 4$$
$$2n^2 + n - 4 = 899$$
$$2n^2 + n - 903 = 0$$
$n = 21$ by quadratic formula

7. Find p if:

a. $2p + 3$; $4p + 2$ and $5p + 3$ are consecutive terms of an arithmetic sequence.

7 ; 10 ; 13

$$T_2 - T_1 = T_3 - T_2$$
$$4p + 2 - (2p + 3) = 5p + 3 - (4p + 2)$$
$$4p + 2 - 2p - 3 = 5p + 3 - 4p - 2$$
$$2p - 1 = p + 1$$
$$p = 2$$

b. $3p + 1$; $2p + 4$ and $p + 8$ are consecutive terms of a geometric sequence.

25; 20; 16

4; 6; 9

$$\frac{T_2}{T_1} = \frac{T_3}{T_2}$$
$$\frac{2p+4}{3p+1} = \frac{p+8}{2p+4}$$
$$p - 2 = 7$$
$$3p = 7$$
$$3p^2 + 24p + p + 8 = 4p^2 + 8p + 8p + 16$$
$$0 = p^2 - 9p + 8$$
$$p = 8 \text{ or } 1$$

c. $p - 1$; $2p - 1$; $4p - 2$ and $5p$ are consecutive terms of a quadratic progression

$$2p - 1 - p + 1 \ ; \ 4p - 2 - 2p + 1 \ ; \ 5p - 4p + 2$$
$$p \ ; \ 2p - 1 \ ; \ p + 2 \quad \leftarrow 1^{st} \text{ differences}$$
$$2p - 1 - p = p + 2 - 2p + 1$$
$$p - 1 = -p + 3$$
$$2p = 4$$
$$p = 2$$

$\boxed{1;\ 3;\ 6;\ 10}$

8. A radioactive substance weighing 8 kg loses 9% of its mass each week.
After how many weeks will it first weigh less than a kilogram?

After 1 week $= 8 \times 0{,}91$

2 weeks $= 8 \times 0{,}91^2$

3 weeks $= 8 \times 0{,}91^3$

$$8 \times 0{,}91^n = 1$$
$$0{,}91^n = \frac{1}{8}$$
$$n = \log_{0{,}91} \frac{1}{8}$$
$$n = 22{,}04$$
$\therefore$ 23 weeks

9. A tree grows 1,2 m in its first year. Each year thereafter it grows by 20%

$\therefore 23$ weeks

9. A tree grows 1,2 m in its first year. Each year thereafter it grows by 20% of the amount it grew the year before. What is the maximum height it will ever reach?

$1{,}2 \; ; \; 1{,}2(0{,}2) \; ; \; (1{,}2)(0{,}2^2) \; ; \ldots$

Geometric $r = 0{,}2$ $\quad -1 < r < 1$

$$S_\infty = \frac{1{,}2}{1-0{,}2} = \frac{1{,}2}{0{,}8}$$

$$= 1{,}5\text{ m}$$

10. For what value(s) of k will $\sum_{i=1}^{\infty} (-3k+4)^i$ converge?

$(-3k+4) + (-3k+4)^2 + (-3k+4)^3 + \ldots$

$r = -3k+4$ conv. when $-1 < r < 1$

$-1 < -3k + 4 < 1$

$-5 < -3k < -3$

$5/3 > k > 1$ $\quad \therefore \; 1 < k < \frac{5}{3}$

11. The first differences of a quadratic sequence are given by the formula $T_n = 2n + 2$. Find the 20th term of the quadratic sequence if the 10th term is 111.

1st differences are $4; 6; 8; 10; \ldots$

second differences $2; 2; 2; \ldots$

$a = 1 \quad 3(1) + b = 4 \quad \therefore b = 1$

$T_n = n^2 + n + c$

$T_{10} = 10^2 + 10 + c = 111$

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$\therefore c = 1$

$\therefore T_n = n^2 + n + 1 \quad T_{20} = 421$

Arithmetic ? X
Geometric ? X

12. Which term in the sequence 5 ; 12 ; 23 ; 38 ; 57 ; .... is equal to 597?

7 ; 11 ; 15 ; 19
4 ; 4 ; 4 ; 4

$a = \frac{4}{2} = 2$

$3(2) + b = 7$

$b = 1$

$\boxed{T_n = 2n^2 + n + 2}$

$2 + 1 + c = 5$

$c = 2$

6/7

$597 = 2n^2 + n + 2$

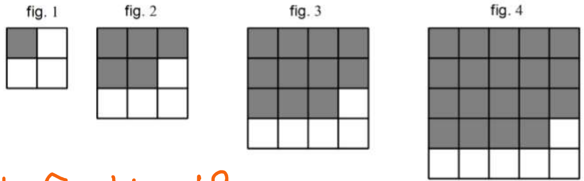
$0 = 2n^2 + n - 595$

$n = \frac{-1 \pm \sqrt{1^2 - 4(2)(-595)}}{4}$

$n = 17$ or $\cancel{-\frac{35}{2}}$

13. Which figure will have 1891 shaded squares?

fig. 1 fig. 2 fig. 3 fig. 4



1 ; 5 ; 11 ; 19 ; . . . . . .
4 ; 6 ; 8 ; . . . 1st differences
2 2 2nd differences

$a = 1$

$3(1) + b = 4$ $b = 1$

$1 + 1 + c = 1$ $\therefore c = -1$

$T_n = n^2 + n - 1$

$n^2 + n - 1 = 1891$

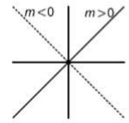
$n^2 + n - 1892 = 0$

$n = 43$ by quadratic formula

Paper 1 – Functions and Inverses

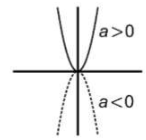
The Linear function – straight line
Standard form: $y = mx + c$

Increasing if $m > 0$
Deceasing if $m < 0$



The Quadratic function – parabola
Parent function: $y = ax^2$
Standard form: $y = ax^2 + bx + c$
Axis of symmetry: $x = -\dfrac{b}{2a}$

"happy" if $a > 0$ and "sad" if $a < 0$

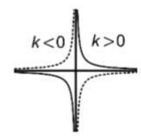


The hyperbolic function – hyperbola

Parent function: $y = \dfrac{k}{x}$

Axes of symmetry are $y = x$ and $y = -x$

Asymptotes are $y = 0$ and $x = 0$
Top right & bottom left if $k > 0$
Top let and bottom right if $k < 0$

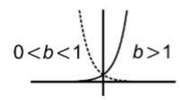


Standard form: $y = \dfrac{k}{x + p} + q$

The exponential function – exponential

Parent function: $y = ab^x$

Asymptote is $y = 0$
Decreasing if $0 < b < 1$
Increasing if $b > 1$

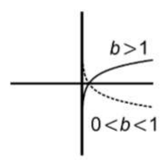


Standard form: $y = ab^x + c$

The logarithmic function – log

Parent function: $y = \log_b x$
Standard form: $y = \log_b(x + p) + q$

Asymptote is $x = 0$



Transformations

If, for example, $f(x) = x^2 + 2x + 6$ then:

$f(-x) = (-x)^2 + 2(-x) + 6$	**reflects f in the y-axis**
$-f(x) = -(x^2 + 2x + 6)$	**reflects f in the x-axis**
$f(x + p) = (x + p)^2 + 2(x + p) + 6$	**shifts f left ($p > 0$) or right ($p < 0$)**
$f(x) + q = x^2 + 2x + 6 + q$	**shifts f up ($q > 0$) or down ($q < 0$)**
$f^{-1}(x)$	**reflects in the line $y = x$**

Questions on Functions and Inverses

x uniquely defines y
for an x only 1 y

1. For each of the following say, with justification, whether we have a function.

a. (-2;5) (1;3) (7;8) (1;5) (9;11)

No, x does not give only 1 y

b. $g(x) = \pm\sqrt{x + 4}$

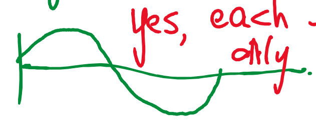
No 5 gives 2 ys

(5; 3)
(5; -3)

c. (-3;1) (2;5) (4;1) (5;6) (7;9)

Yes each x only gives 1 y

d. $y = \sin x$



yes, each x only 1 y

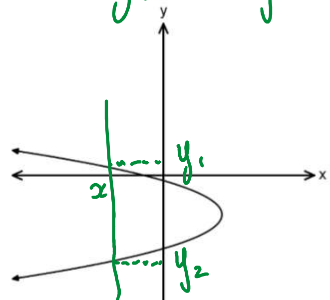
a.

y

f.

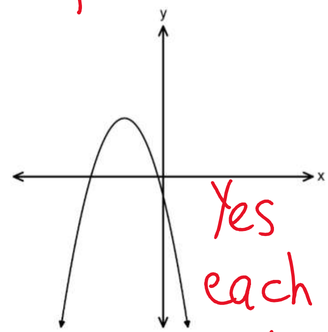
y

a. gives 1 y



No, each x gives more than 1 y

f.



Yes each x only 1 y

2. Give the domain and range of each of the following functions:

a. $y = \frac{12}{x-4} + 3$

Hyperbola

$x \neq 0$
$y \neq 0$

Dom : $x \neq 4$

Range : $y \neq 3$

b. $y = \log_2(x+4)$

4 left

$x + 4 > 0$

D : $x > -4$ R : $y \in \mathbb{R}$

$y = \log_2 x$

Dom: $x > 0$

Range: $y \in \mathbb{R}$

c. $y = 0.5^x - 9$

Down 9

Dom: $x \in \mathbb{R}$

Range: $y > -9$

Dom : $x \in \mathbb{R}$

Range : $y > 0$

d. $y = -(x+4)^2 + 9$

4 units left

9 units up

$(-4;9)$

$(0;0)$

Dom : $x \in \mathbb{R}$

Range : $y \leq 9$

Domain $x \in \mathbb{R}$

Range $y \leq 0$

3. Find the turning point and the range of the following functions by completing the square:

a. $y = x^2 + 6x + 11$

$= (x+3)^2 + 11 - 9$

$y = (x+3)^2 + 2$

Moved 3 left

2 up

T.P. $(-3;2)$

$y = x^2$

$(0;0)$

Range $y \geq 2$

b. $y = -x^2 + 8x - 6$

$= -[x^2 - 8x + 6]$

$= -[(x-4)^2 + 6 - 16]$

$= -[(x-4)^2 - 10]$

$= -(x-4)^2 + 10$

4 right 10 up

T.P. $(4;10)$

$(0;0)$

$y = -x^2$

$(a+b)^2$

$= a^2 + 2ab + b^2$

$(\triangle + \infty)^2$

$= \triangle^2 + 2\triangle\infty + \infty^2$

Range

$y \leq 10$

4. Draw graphs of the following functions showing all points of interest. Where there are asymptotes, they should be drawn (with dotted lines) and labelled.

a. $y = \frac{-9}{x+4} - 2$

Hyperbola

$\frac{2}{3}$ | $\frac{1}{4}$

Moved 4 Left

2 down

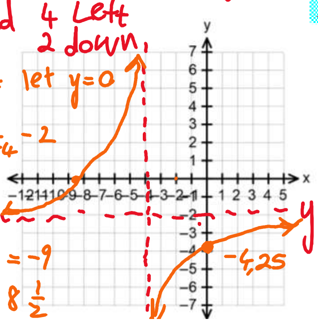
x-int let $y = 0$

$0 = \frac{-9}{x+4} - 2$

$2 = \frac{-9}{x+4}$

$2(x+4) = -9$

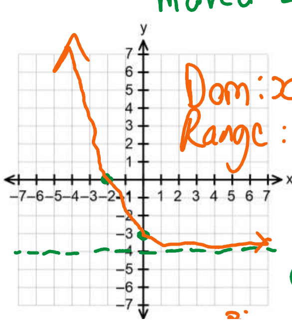
$x = -8\frac{1}{2}$



b. $y = 0.5^x - 4$

exponential

moved 4



Dom: $x \in \mathbb{R}$

Range : $y > -4$

y-int let $x = 0$

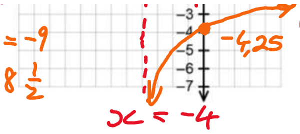
$y = -3$

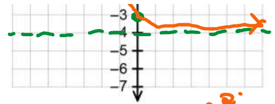
x-int let $y = 0$

$0 = \left(\frac{1}{2}\right)^x - 4$

$4 = \left(\frac{1}{2}\right)^x$

$2(x+4)=-9$
$x=-8\frac{1}{2}$



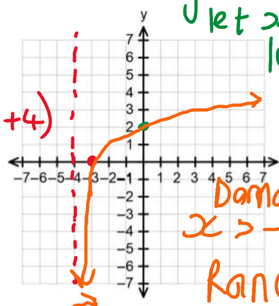


$4=\left(\frac{1}{2}\right)^{x}$
$x=\log_{\frac{1}{2}}4$
$x=-2$

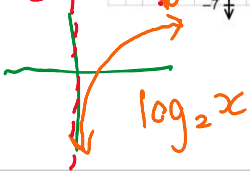
c. $y=\log_2(x+4)$

x-int
let $y=0$
$0=\log_2(x+4)$
$x+4=2^0$
$x+4=1$
$x=-3$

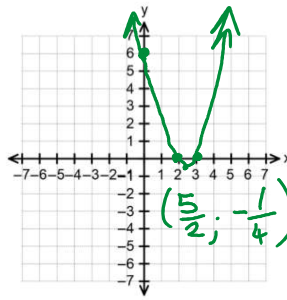
y-int
let $x=0$
$\log_2 4=4$



Domain
$x>-4$
Range
$y\in R$



d. $y=x^2-5x+6$



y-int $(0;6)$
x-ints let $y=0$
$x^2-5x+6=0$
$(x-3)(x-2)=0$
$x=3$ or 2

$x=-\frac{b}{2a}$
$x=\frac{5}{2}$

Dom: $x\in R$
Range: $y\geq-\frac{1}{4}$

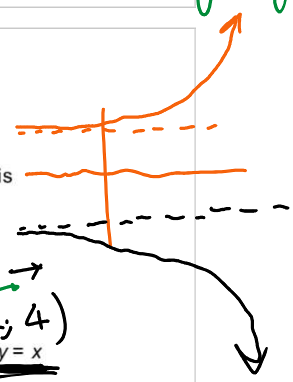
5. Give, in standard form, the equation of the function which results when:

a. $y=x^2+3x-5$
is shifted 3 left and 2 up

$y=(x+3)^2+3(x+3)-5+2$

b. $y=3^x+5$
is reflected in the x-axis

$y=-(3^x+5)$
$y=-3^x-5$



b. $y=\frac{7}{x+3}-2$
is reflected in the y-axis

$y=\frac{7}{-x+3}-2$

d. $y=3^x-5$
is reflected in the line $y=x$

$(2;4)$

$x=3^y-5$
$x+5=3^y$
$y=\log_3(x+5)$

$(4;2)$

6. Find the inverses of the following functions in the form:

a. $f(x)=3x-5$

$y=3x-5$
$x=3y-5$
$x+5=3y$
$y=\frac{x+5}{3}$
$f^{-1}(x)=\frac{x+5}{3}$

b. $f(x)=2\times3^x+6$

$y=2\times3^x+6$
$x=2\times3^y+6$
$\frac{x-6}{2}=3^y$
$y=\log_3\frac{x-6}{2}$
$f^{-1}(x)=\log_3\left(\frac{x-6}{2}\right)$

c. $f(x)=\log_2(x+3)-5$

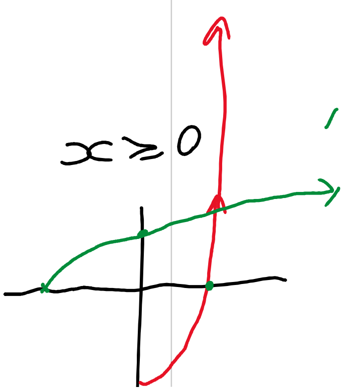
$(5;-2)$

$y=\log_2(x+3)-5$
$x=\log_2(y+3)-5$
$x+5=\log_2(y+3)$
$2^{x+5}=y+3$

d. $f(x)=x^2-4$
give a domain restriction which will ensure that $f^{-1}(x)$ is a function

$y=x^2-4$
$x=y^2-4$
$x+4=y^2$

$x\geq0$



$2^{x+5} = y+3$ $(-2;5)$

$2^{x+5} - 3 = y$

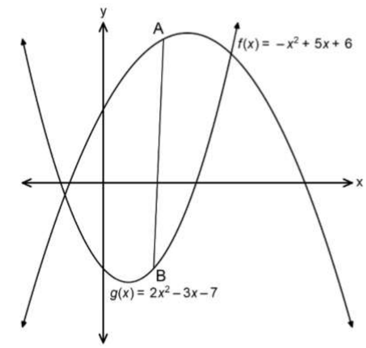
$f^{-1}(x) = 2^{x+5} - 3$

$x+4 = y^2$

$\sqrt{x+4} = y$

$f^{-1}(x) = \sqrt{x+4}$

7. Consider the sketch below. AB is a vertical line with A on *g* and B on *f*.



can also find x of T.P. using $x = -\frac{b}{2a}$

Find the maximum length of AB if it is on the interval where *g* exceeds *f*.

$AB = -x^2 + 5x + 6 - (2x^2 - 3x - 7)$

$AB = -3x^2 + 8x + 13$

$AB = -3\left[x^2 - \frac{8}{3}x - \frac{13}{3}\right]$

$AB = -3\left[\left(x - \frac{4}{3}\right)^2 - \frac{16}{9} - \frac{39}{9}\right] = -3\left(x - \frac{4}{3}\right)^2 + \frac{55}{3}$

Max length $= \frac{55}{3}$ when $x = \frac{4}{3}$

8. Find the equations of the axes of symmetry of $y = -\frac{12}{x+3} - 4$

axes of symmetry of $y = \frac{-12}{x}$ are $y = x$ and $y = -x$

but they must both be moved 3 left and 4 down

$y = x + 3 - 4$ and $y = -(x+3) - 4$

$y = x - 1$ and $y = -x - 7$

OR find equations of lines with gradients of $+1$ and -1 passing through pt. of intersection of asymptotes $(-3;-4)$

9. For what value(s) of *k* will $y = x^2 + 3x - 3$ and $y = 2x + k$ not intersect one another?

$x^2 + 3x - 3 = 2x + k$

$x^2 + x - 3 - k = 0$

$\Delta = 1^2 - 4(1)(-3-k) = 13 + 4k < 0$

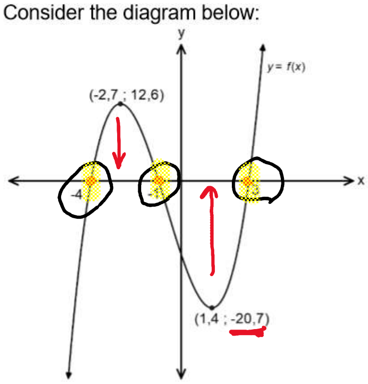
$x^2 + x - 3 - k = 0$

$\Delta = 1^2 - 4(1)(-3-k) = 13 + 4k < 0$

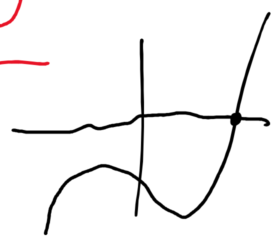
$4k < -13$

$k < -\frac{13}{4}$

10. Consider the diagram below:



$f(x) = 0$



a. For what value(s) of t will $f(x) + t = 0$ have exactly one root?

$t > 20,7 \qquad t < -12,6$

b. For what value(s) of k if will $f(x + k) = 0$ have only positive roots?

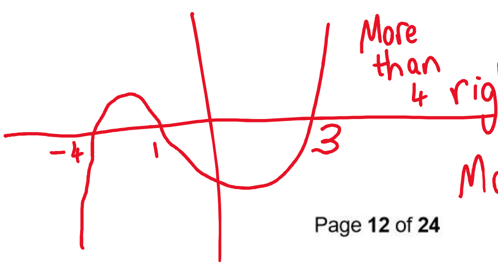
more than 4 right

$k < -4$

c. For what value(s) of t will $f(x) + t = 0$ have three distinct roots?

$-12,6 < t < 20,7$

d. For what value(s) of k if will $f(x + k) = 0$ have 3 roots of the same sign?



More than 4 right

$k < -4$

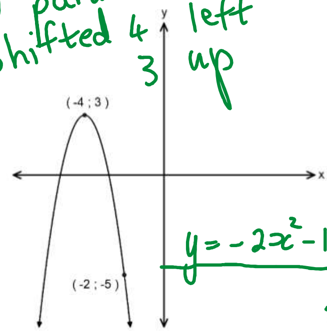
More than 3 left

$k > 3$

11. Find the equations of the following functions:

a.

A parabola $y = ax^2$
shifted 4 left
3 up



$y = a(x+4)^2 + 3$

Subst $(-2; 5)$

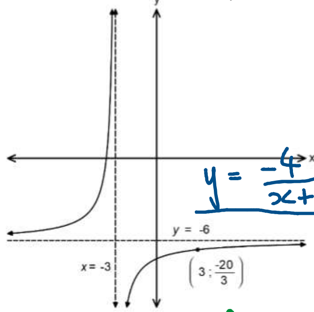
$-5 = a(-2+4)^2 + 3$

$-8 = 4a \qquad a = -2$

$y = -2x^2 - 16x - 29$

b.

A hyperbola $y = \frac{k}{x}$
shifted 3 left
6 down



$y = \frac{k}{x+3} - 6$

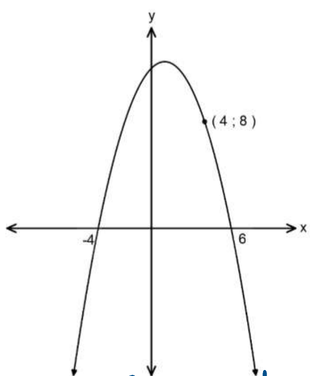
Subst $(3; -\frac{20}{3})$

$-\frac{20}{3} = \frac{k}{6} - 6$

$-\frac{2}{3} = \frac{k}{6} \qquad \therefore k = -4$

$y = \frac{-4}{x+3} - 6$

c.



roots of -4 and 6

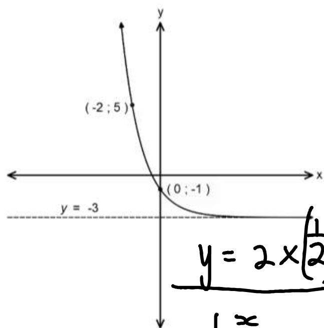
$\therefore y = a(x+4)(x-6)$

Subst. $(4; 8)$

$8 = a(8)(-2) \qquad \therefore a = -\frac{1}{2}$

$y = -\frac{1}{2}x^2 + x + 12$

d.



$y = ab^x + c$

$y = ab^x - 3$

subst $(0; -1)$

$-1 = ab^0 - 3$

$a = 2$

$y = 2b^x - 3$

subst $(-2; 5)$

$5 = 2b^{-2} - 3$

$8 = \frac{2}{b^2} \qquad \therefore b^2 = \frac{2}{8} = \frac{1}{4}$

$\therefore b = \frac{1}{2}$

$y = 2 \times \left(\frac{1}{2}\right)^x - 3$

PAPER 2 - Trigonometry

Not provided:

$\sin\theta = \frac{y}{r}$ or $\frac{\text{opp}}{\text{hyp}}$ $\qquad$ $\cos\theta = \frac{x}{r}$ or $\frac{\text{adj}}{\text{hyp}}$ $\qquad$ $\tan\theta = \frac{y}{x}$ or $\frac{\text{opp}}{\text{adj}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ $\cos^2\theta = 1 - \sin^2\theta$ $\qquad$ $\sin^2\theta = 1 - \cos^2\theta$

Provided:

In Δ ABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc\cos A$

In Δ ABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha\cos\alpha$

1. Simplify without a calculator:

$\boxed{30^\circ \; 45^\circ \; 60^\circ}$

$\cos 15^\circ = \cos(45^\circ - 30^\circ)$

$= \cos 45 \cos 30 + \sin 45 \sin 30$

$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} = \frac{\sqrt{6}+2}{4}$

$\cos 195^\circ \sin 285^\circ + \cos 105^\circ \sin 15^\circ$

$= -\cos 15^\circ \times -\cos 15^\circ - \sin 15^\circ \sin 15^\circ$

$= \cos^2 15 - \sin^2 15 = \cos 2(15) = \cos 30^\circ$

$= \frac{\sqrt{3}}{2}$

S | A
T | C

$\frac{\sin 140^\circ \cos 240^\circ}{\sin 340^\circ \sin 110^\circ \cos 180^\circ}$

$= \frac{\sin 40^\circ \times -\cos 60}{-\sin 20^\circ(\cos 20^\circ) \times -1}$

$= \frac{\sin 40^\circ \times -\frac{1}{2}}{-\frac{1}{2}\sin 40^\circ \times -1} = -1$

S | A
T | C

2. If $\cos 13^\circ = m$ then give the following in terms of m:

$\cos 193^\circ$

$= \cos(180 + 13)$

$= -\cos 13^\circ$

$= -m$

$\sin 283^\circ$

$= \sin(270 + 13^\circ)$

$= -\cos 13^\circ$

$= -m$

$\sin(-347^\circ)$

$= -\sin 347^\circ$

$= -\sin(360 - 13)$

$= -(-\sin 13)$

$= \sin 13^\circ$

$= \sqrt{1-m^2}$

$\sin 13^\circ$

$\cos 386^\circ$

$= \cos(360^\circ + 26^\circ)$

$= \cos 26^\circ$

$= \cos 2(13^\circ)$

$= 2\cos^2 13^\circ - 1 = 2m^2 - 1$

$\cos 6.5^\circ$

$\cos 13 = \cos 2(6,5)$

$m = 2\cos^2 6,5 - 1$

$m + 1 = 2\cos^2 6,5$

$\frac{m+1}{2} = \cos^2 6,5$

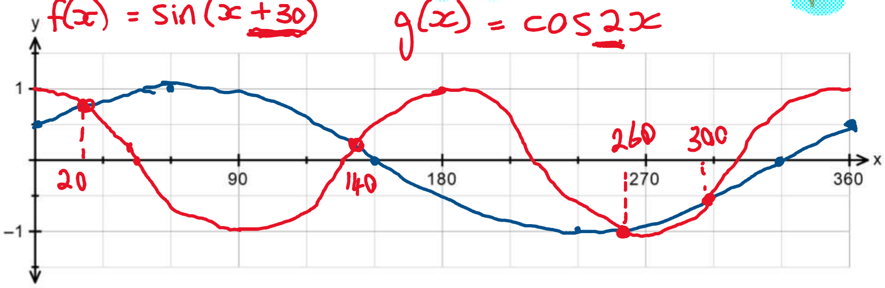
$\sqrt{\frac{m+1}{2}} = \cos 6,5$

1, 13°, m, $\sqrt{1-m^2}$

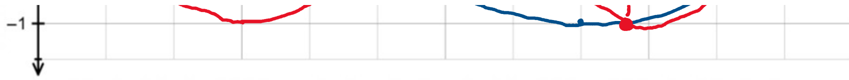
P

3. Draw on the same set of axes:
$f(x) = \sin x \cos 30^\circ + \cos x \cos 30^\circ$ and $g(x) = \cos^2 x - \sin^2 x$

$f(x) = \sin(x + 30)$ $\qquad$ $g(x) = \cos 2x$



$\sin 2x = \cos 2x$



Calculate the points of intersection of the two graphs:

$\sin(x+30^\circ) = \cos 2x$

$\cos(90-(x+30)) = \cos 2x$

$\cos(60-x) = \cos 2x$

Q1 $\quad 60-x = 2x+360k$

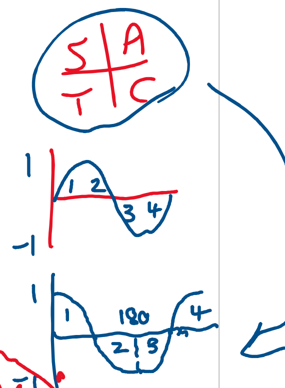
$-3x = -60+360k$

$x = 20-120k$

$20^\circ;\ 140^\circ;\ 260^\circ;\ 300^\circ$

Q4 $\quad 60-x = 360-2x+360k$

$x = 300^\circ+360k$



4. Solve $\cos 2\theta \cos 10° + \sin 2\theta \sin 10° = -\frac{\sqrt{3}}{2}$ for $\theta \in [0°;360°]$

$$\cos(2\theta - 10°) = -\frac{\sqrt{3}}{2}$$

$$\text{ref } \angle = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$$

2 ✓ 1
3 ✓ 4

$$2\theta - 10° = 180° - 30° + 360k$$

$$2\theta - 10° = 180° + 30° + 360k \qquad k \in Z$$

5. Prove:

a. $\frac{\sin 3x}{\sin x} + \frac{\cos 3x}{\cos x} = 4\cos 2x$

$$LHS = \frac{\sin 3x \cos x + \cos 3x \sin x}{\sin x \cos x}$$

$$= \frac{\sin(3x + x)}{\sin x \cos x} = \frac{2\sin 4x}{2\sin x \cos x} = \frac{2(\sin 2(2x))}{\sin 2x}$$

$$= \frac{2(2\cancel{\sin 2x}\cos 2x)}{\cancel{\sin 2x}}$$

$$= 4\cos 2x$$

$$= RHS$$

b. $\frac{2\sin^2\theta}{1+\cos 2\theta} = \tan^2\theta$

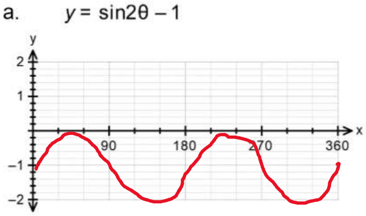
$$LHS = \frac{2\sin^2\theta}{1 + 2\cos^2\theta - 1}$$

$$= \frac{\cancel{2}\sin^2\theta}{\cancel{2}\cos^2\theta}$$

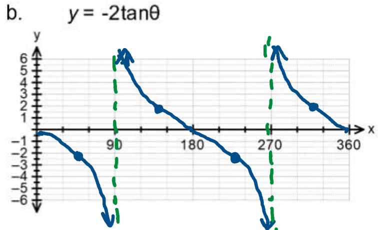
$$= \left(\frac{\sin\theta}{\cos\theta}\right)^2 = \tan^2\theta = RHS$$

6. Sketch the following graphs on the axes provided:

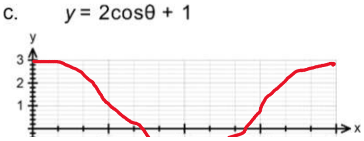
a. $y = \sin 2\theta - 1$



b. $y = -2\tan\theta$

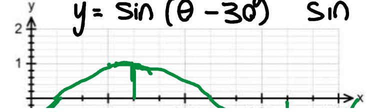


c. $y = 2\cos\theta + 1$

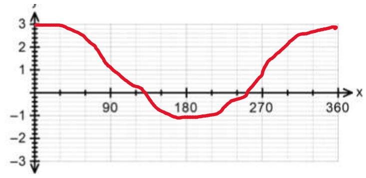


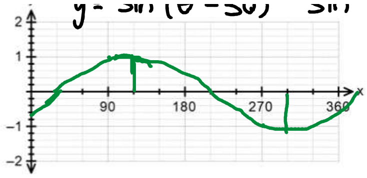
d. $y = \sin\theta\cos 30° - \cos\theta\sin 30°$

$y = \sin(\theta - 30°)$ sin moved right 30°



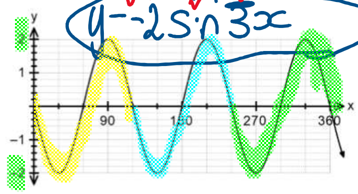
$y = \sin(\theta - 30)$ sin moved right 30°



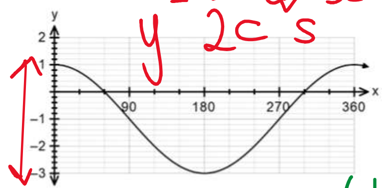


7. Find the equations of the following functions:

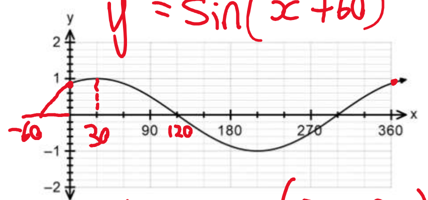
a. $y = -2\sin 3x$



b. $y = 2\cos x$

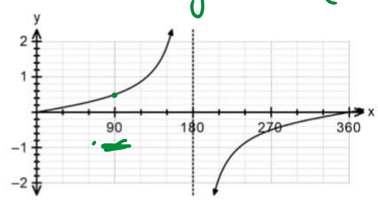


c. $y = \sin(x + 60)$

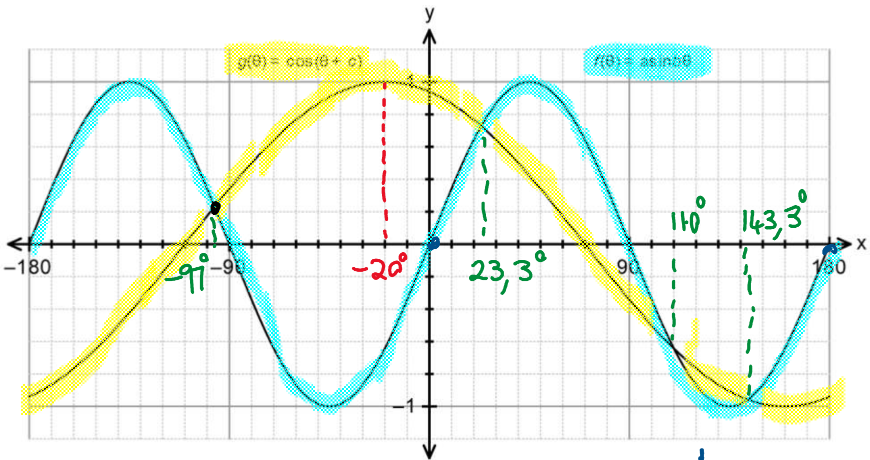


$y = \cos(x - 30)$

d. $y = \frac{1}{2}\tan\left(\frac{1}{2}x\right) = \frac{1}{2}\tan\left(\frac{x}{2}\right)$



8. Consider the following functions drawn below for θ∈[-180°;180°]



a. Determine the values of *a*, *b* and *c*.

$a = 1$ $b = 2$

$c = 20°$

b. What is the period of *f* ?

180°

c. What is the amplitude of *f* ?

1

d. Determine the *x*-values of the points of intersection on the given domain.

$$\cos(\theta + 20°) = \sin 2\theta$$

$$\cos(\theta + 20°) = \cos(90° - 2\theta)$$

$$\theta + 20° = 90 - 2\theta + 360k \qquad \theta + 20 = 360° - (90 - 2\theta) + 360k$$

$$\theta = \frac{70}{3} + 120k \qquad \theta = -250 - 360k$$

e. Give the value(s) of θ on the given domain for which $g(\theta) > f(\theta)$

23,3° or 143,3°

110° or −96,7°

$-96{,}7° < \theta < 23{,}3°$

or

$110° < \theta < 143{,}3°$