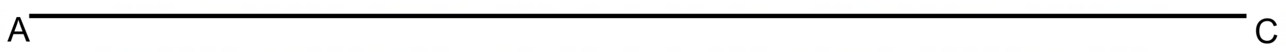
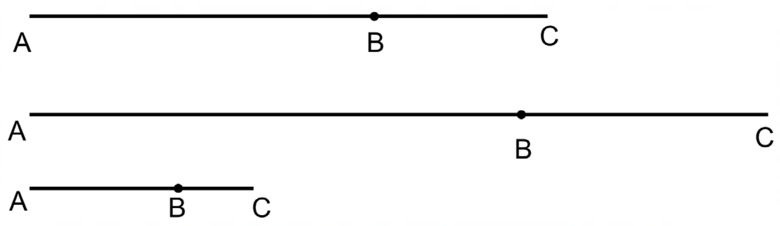
Similarity and Proportion

Ratio is central to the study of similarity and proportion.

Place a point B on this line to satisfy $AB : BC = 2 : 1 \quad OR \quad \frac{AB}{BC} = \frac{2}{1}$



Any of these lines meet the requirement:



Ratio has to do with the relationship between items rather than the actual length.

Remember too that the same rules which apply to fractions apply to ratios. We generate equivalent ratios by ________________________ or ________________________ both sides (all three sides) by ______________________________.

2 : 5

= ____ : 10

= 8 : ____

2 : 3 : 7

= 4 : ____ : ____

= ____ : 15 : ____

= ____ : ____ : 70

Another cool time-saving trick….

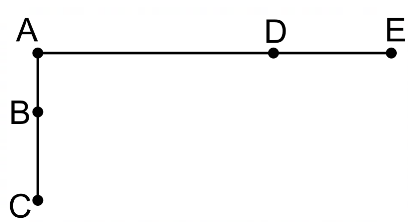
If $\frac{a}{b}=\frac{c}{d}$ then multiplying both sides by b gives: $a=\frac{bc}{d}$ and multiplying both sides by d gives: $ad=bc$

In other words, if two fractions are equal then

If we are given the ratio of two lengths, then it is **not** a good idea to simply fill in the values on the diagram as the following example will show:

$AD : DE = 2 : 1$

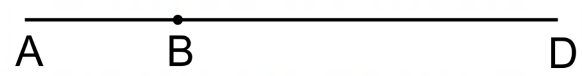
$\frac{AB}{BC}=\frac{1}{2}$



We may be misled into thinking that $AB = DE$ which it clearly is not!

However, we can overcome this by assigning lengths using a variable. Let's demonstrate this with an example:

Suppose $AB : BD = 2 : 5$. Find $AB : AD$



Let's use this strategy to answer this question.

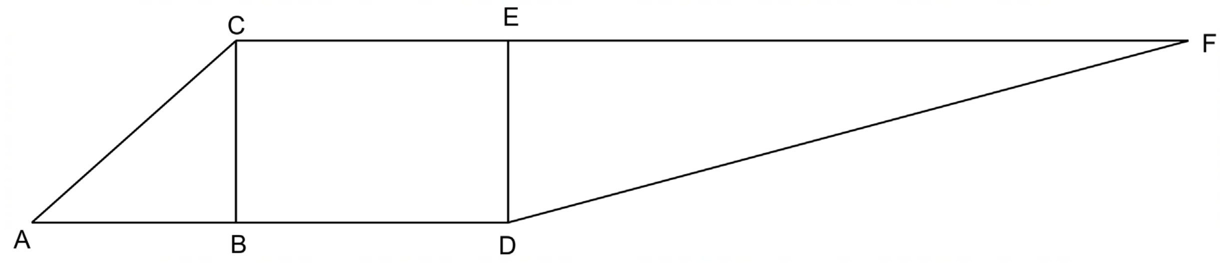
Four points A, B, C and D are on a straight line such that $\frac{AB}{BC} = \frac{2}{3}$ and $\frac{BC}{CD} = \frac{6}{7}$.

Determine $\frac{AB}{CD}$ and $\frac{AB}{AD}$.

$\frac{AB}{CD}$

$\frac{AB}{AD}$

We can do something similar with areas.



If $\Delta ABC : BCED = 3 : 8$ and $BCED : \Delta DEF = 2 : 3$ then determine:

$\Delta ABC : \Delta DEF$

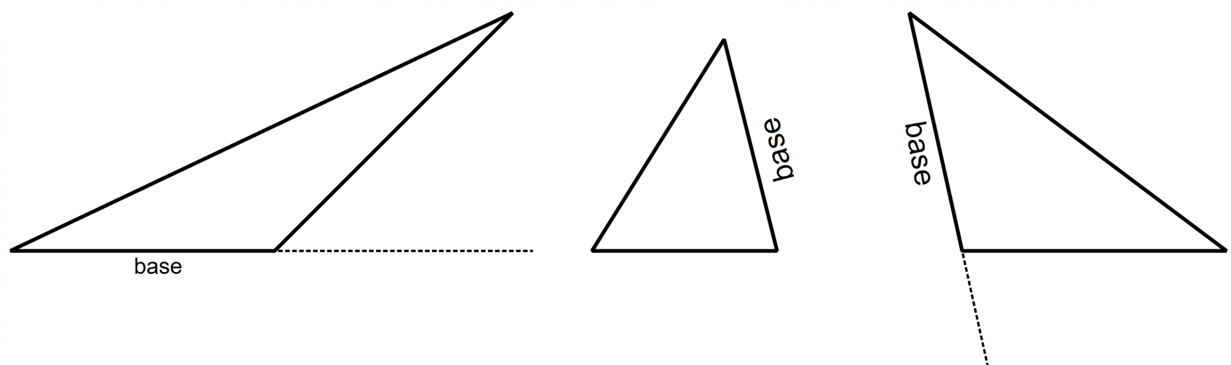
ABDEC : ΔDEF

The 4 big ideas!

Next up we are going to look at three results concerning the areas of triangles. Remember that the area of a triangle is $\frac{1}{2} \times base \times height$

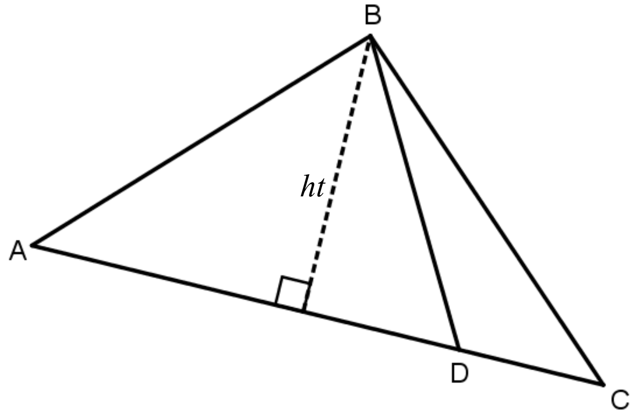
The height of the triangle is the height of the highest point vertically above the base. This means that a triangle has three heights – one for each of its bases.

Draw in the heights of the given triangles for each of the bases which have been labelled:



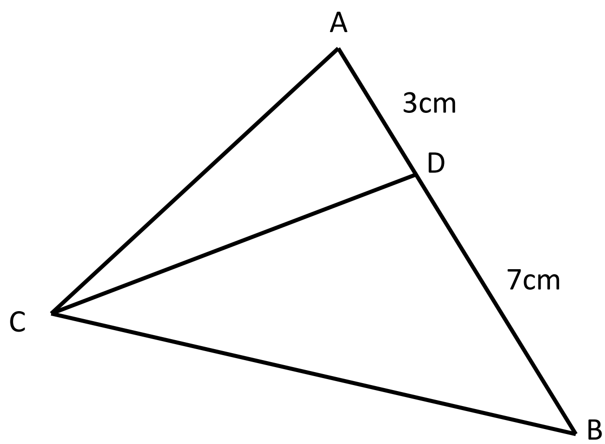
BIG IDEA 1

The areas of triangles with a common height are in the same ratio as their bases (common / shared ht)

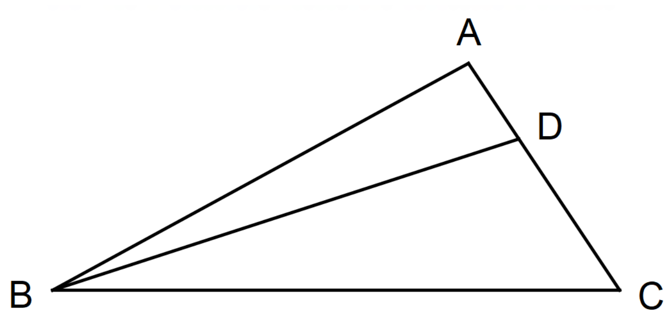


$$\frac{\text{area } \Delta BAD}{\text{area } \Delta BDC} = \frac{\frac{1}{2} \times AD \times ht}{\frac{1}{2} \times DC \times ht} = \frac{AD}{DC}$$

1. Calculate the ratio of $\frac{\text{area } \Delta ACD}{\text{area } \Delta ABC}$, giving a reason for your answer:



2. If $\frac{AD}{DC} = \frac{1}{2}$ then determine $\frac{\text{area } \Delta ABD}{\text{area } \Delta ABC}$ and $\frac{\text{area } \Delta BDC}{\text{area } \Delta ABD}$ giving reasons

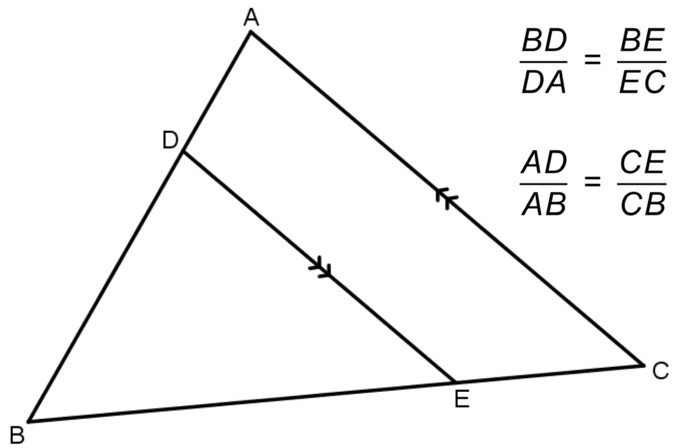


$\frac{\text{area } \Delta ABD}{\text{area } \Delta ABC}$

$\frac{\text{area } \Delta BDC}{\text{area } \Delta ABD}$

BIG IDEA 2

A line drawn parallel to one side of a triangle cuts the other two sides in the same proportion (Proportional Intercept Theorem)

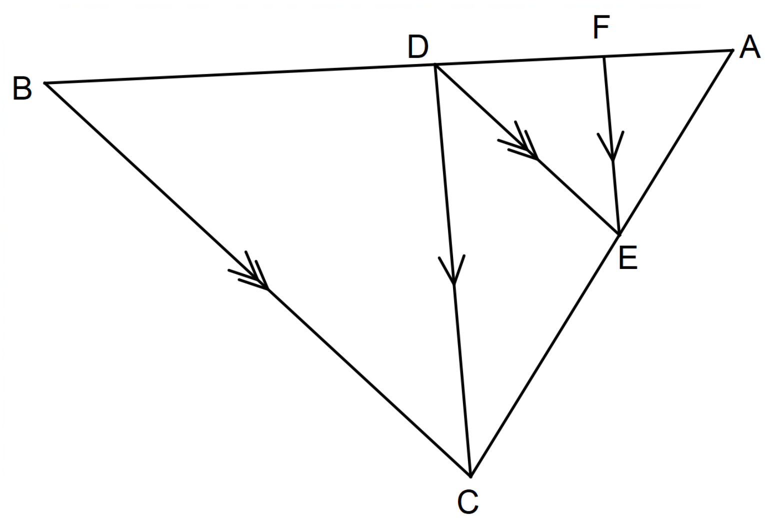


$$\frac{BD}{DA} = \frac{BE}{EC}$$

$$\frac{AD}{AB} = \frac{CE}{CB}$$

The **converse** is true viz.
A line drawn which cuts two sides of a triangle in the same proportion is parallel to the third side (CONVERSE Proportional Intercept Theorem)

7. If $\frac{AF}{FD} = \frac{2}{3}$ then determine $\frac{AF}{DB}$ giving reasons.



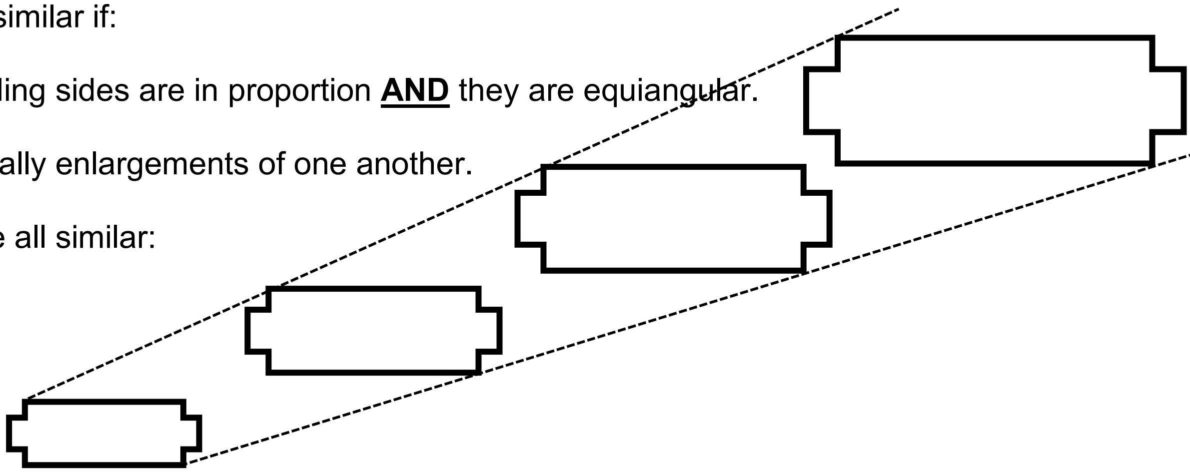
BIG IDEA 3

Two figures are similar if:

Their corresponding sides are in proportion **AND** they are equiangular.

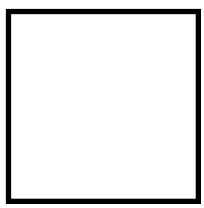
They are essentially enlargements of one another.

The following are all similar:

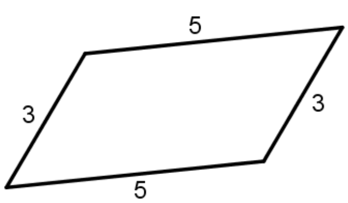


But this one is not!

Simply being equiangular is not enough for two figures to be similar. Consider the two rectangles below:



Likewise, two figures are not similar simply because their corresponding sides are in proportion



HOWEVER, triangles have the property that:

If they are equiangular then the corresponding sides **will** be in proportion.

Likewise,

if the corresponding sides are in proportion, then the triangles **will** be equiangular.

So, for triangles, we have two ways of proving similarity:

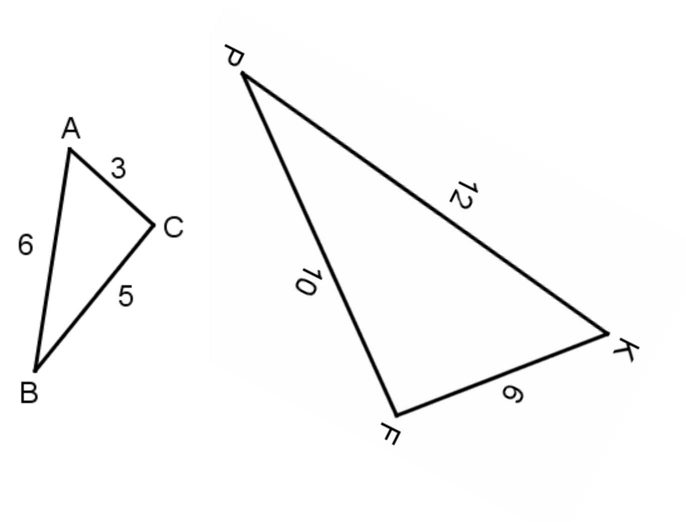
- Prove that the triangles are equiangular

  OR

- Prove that the corresponding sides are in proportion

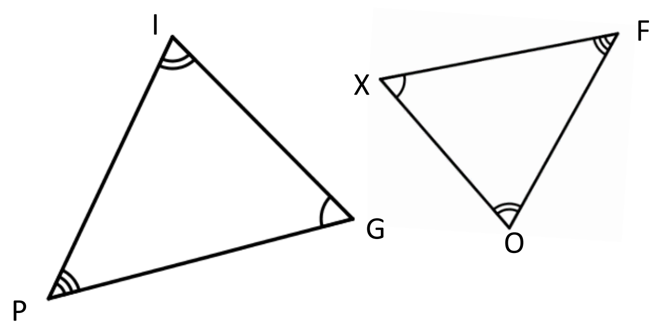
We list them in **corresponding order,** and we use the symbol $|||$ for **similarity**.

Remember that $\equiv$ is the symbol for **congruency**. Congruent means identical and is a much stronger condition than similarity. Congruent figures are always similar but similar figures are not necessarily congruent!



$$\frac{AC}{KF} = \frac{BC}{PF} = \frac{AB}{KP}$$

$\therefore \Delta ABC \ ||| \ \Delta KPF$ (corr. sides in prop.)



$\hat{P} = \hat{F}$ and $\hat{I} = \hat{O}$ and $\hat{G} = \hat{X}$

$\therefore \Delta PIG \ ||| \ \Delta FOX$ (AAA)

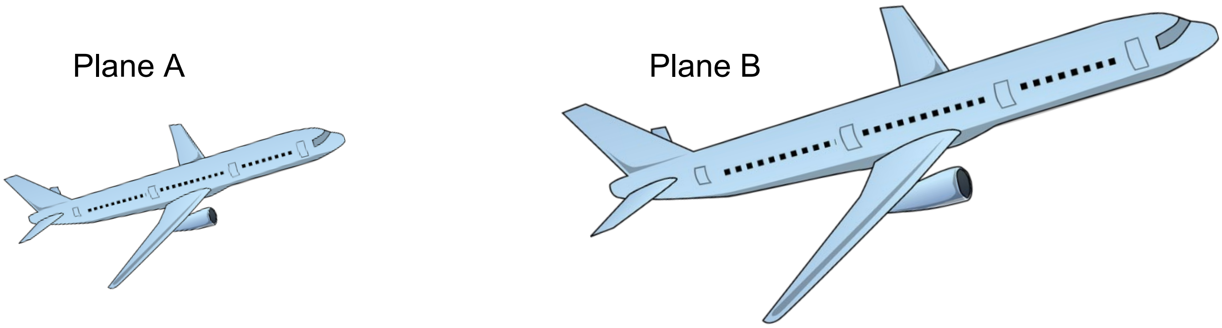
If two shapes are similar, then:

$$\frac{\text{length of piece on first}}{\text{length of corresponding piece on second}} = \frac{\text{length of another piece on first}}{\text{length of corresponding piece on second}}$$

OR

$$\frac{\text{length of piece on first}}{\text{length of another piece on first}} = \frac{\text{length of corresponding piece on second}}{\text{length of corresponding piece on second}}$$

For example: consider these two similar aeroplanes:

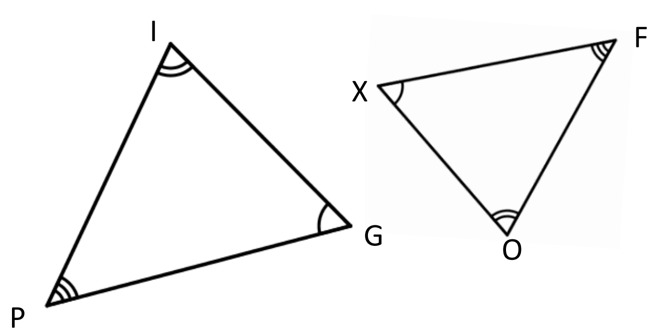


$$\frac{\text{length of plane A}}{\text{length of plane B}} = \frac{\text{wingspan of plane A}}{\text{wingspan of plane B}}$$

OR

$$\frac{\text{height of plane A}}{\text{size of tail of plane A}} = \frac{\text{height of plane B}}{\text{size of tail of plane B}}$$

Likewise, providing we have listed them correctly we can extract ratios from the similarity statement. The reason is **SIMILAR TRIANGLES**.

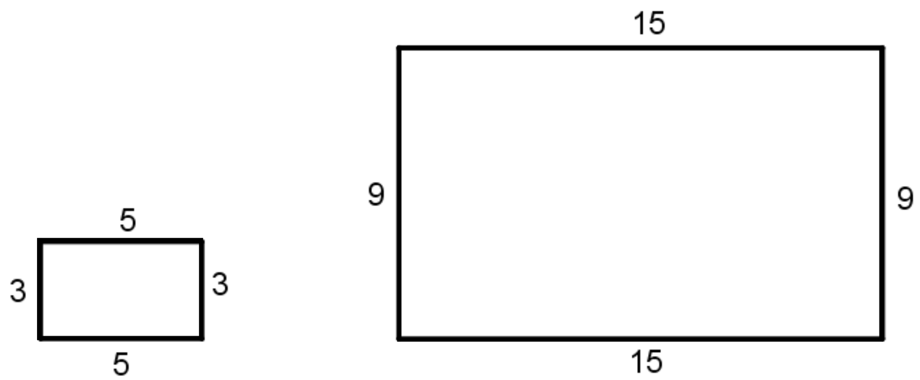


BIG IDEA 4

If two shapes are similar and their sides are in the ratio $a : b$ then their areas will be in the ratio $a^2 : b^2$.

If three dimensional then their volumes will be in the ratio $a^3 : b^3$

This is best understood by considering an example:

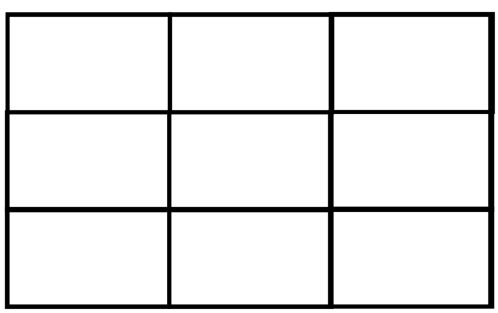


The sides of the rectangle are in the ratio 1:3

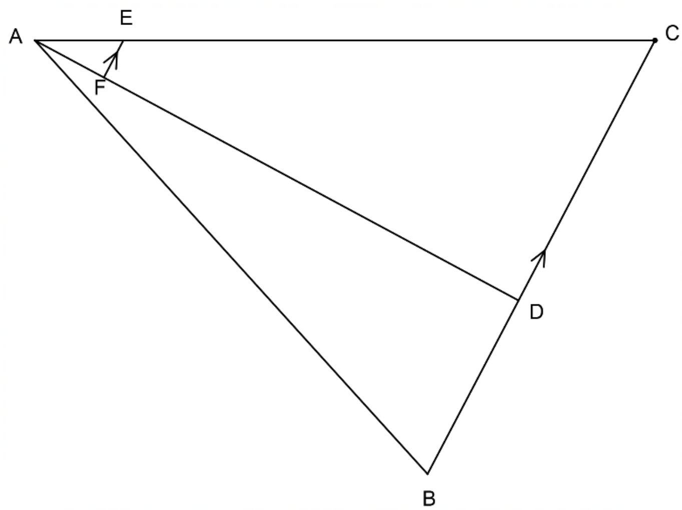
The small rectangle will fit into the big one three times along the width and three times down the height for a total of 9 rectangles.

Since the sides are in the ratio 1:3 the areas are in the ratio $1^2 : 3^2 = 1 : 9$.

The big one is 9 times the size of the little one!



6. In the figure $FE \parallel BC$ and $\frac{AE}{EC} = \frac{1}{6}$ and $BD : BC = 2 : 5$



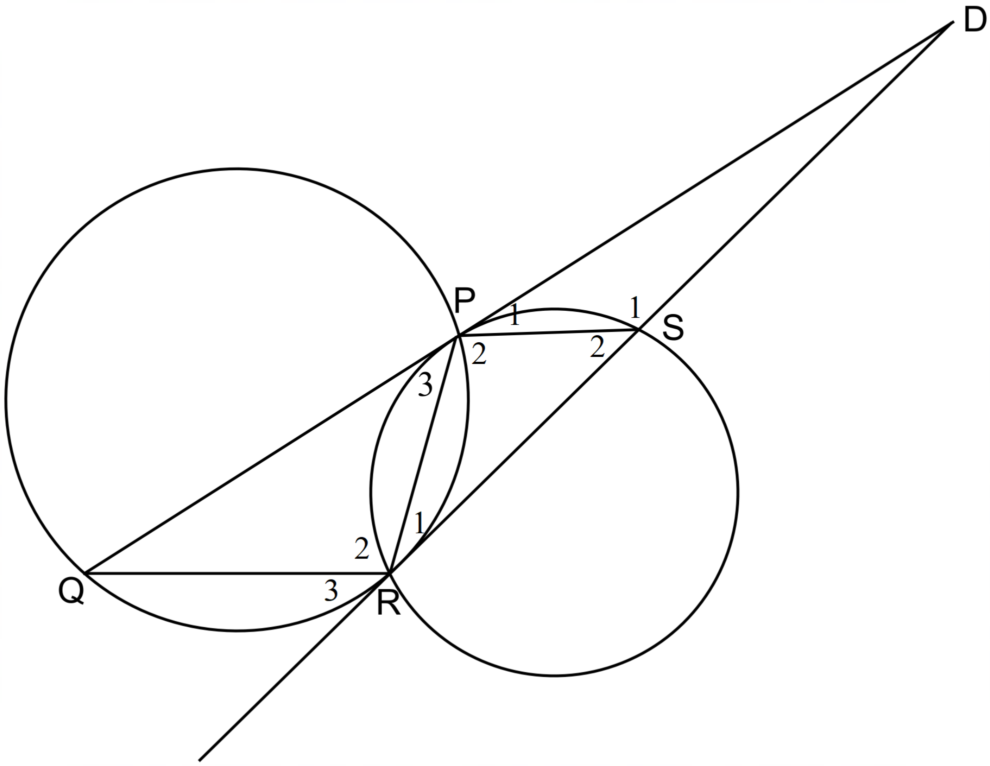
a. Determine $AF : AD$ with a reason.

b. Determine $\frac{\text{area } \Delta ADC}{\text{area } \Delta ABC}$ with a reason.

c. Given that $\Delta AFE \parallel\!| \Delta ADC$ determine $\frac{\text{area } \Delta AFE}{\text{area } \Delta ADC}$.

d. Using your answers to above or otherwise determine: $\frac{\text{area } \Delta AFE}{\text{area } \Delta ABC}$

1. In the accompanying figure, two circles intersect at P and R. DPQ is a tangent to circle PSR, while DSR is a tangent to circle PQR.

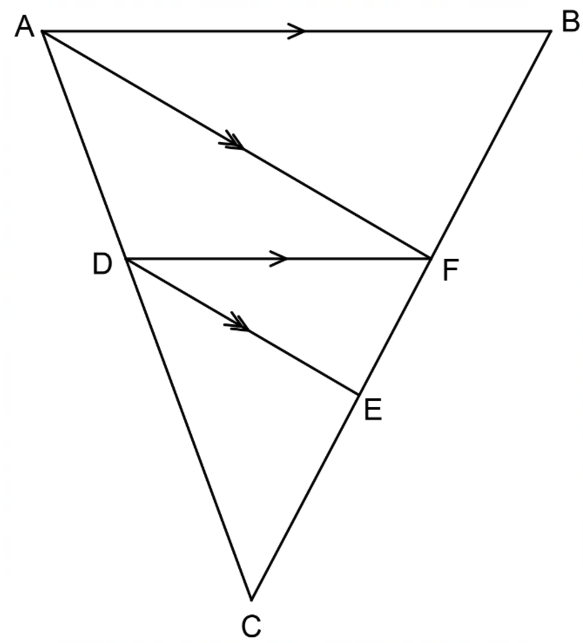


a. Prove that $\Delta DPR \,|||\, \Delta DRQ$

b. Prove that $DR^2 = DQ.DP$

c. Prove that $PS \parallel QR$

15. In the diagram below $AB \| DF$ and $AF \| DE$. $CF : FB = 3 : 2$



Determine, with reasons:

i. $AD : DC$

ii. area ΔAFD : area ΔDFC

iii. $EF : FB$