$f' = 0$

$f' = 0$

OPTIMISATION

To OPTIMISE somethings means to find its BEST value.

This could be the BIGGEST or the SMALLEST depending on the context.

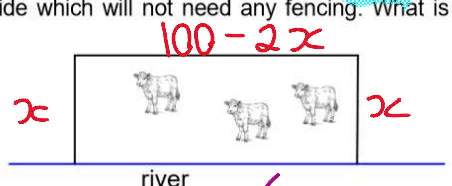
The BIG idea. When a quantity has a maximum or minimum its gradient (derivative) is 0.

EXAMPLE 1	EXAMPLE 2
Find the minimum value of the expression: $C = x^2 - 4x + 8$	Find the maximum value of the expression: $P = -3x^2 + 12x + 40$
$\frac{dC}{dx} = 2x - 4 = 0$ $2x = 4$ $x = 2$ $(2; 4)$	$\frac{dP}{dx} = -6x + 12 = 0$ $-6x = -12$ $x = \frac{-12}{-6} = 2$ $(2; 52)$

So, in order to optimise a quantity we need an expression for it but that expression must only have one variable!

EXAMPLE 3

Suppose a farmer wishes to fence a rectangular field. He has 100 m of fencing but he will use a long straight river for one side which will not need any fencing. What is the largest field he can fence?



$A = 98\ m^2$

$A = 1200\ m^2$

Area $= x(100 - 2x)$

$A = 100x - 2x^2$

$\frac{dA}{dx} = 100 - 4x = 0$

$100 = 4x$

$\frac{dA}{dx} = 0$

A

x

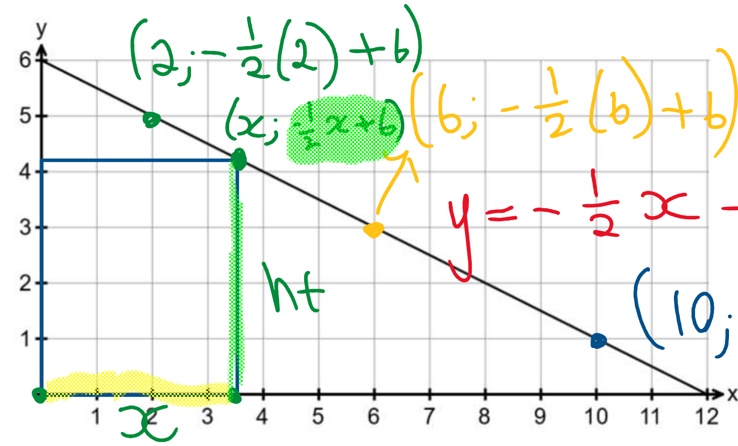
$100 = 4x$

$x = 25$

$A_{max} = 1250 m^2$

EXAMPLE 4

A rectangle is to be drawn so that its bottom left corner is at the origin (0;0) and its top right corner is on the line joining (0;6) to (12;0). What is the biggest area it can have?



$(6; -\frac{1}{2}(6)+6)$

$A = x \cdot ht$

$y = -\frac{1}{2}x + 6$

$(10; -\frac{1}{2}(10)+6)$

gradient $= \frac{\text{rise}}{\text{run}}$

$= \frac{-6}{12}$

$= \frac{-1}{2}$

$A' = 0$?

$A = \text{base} \times ht$

$A = x \times \left(-\frac{1}{2}x + 6\right)$

$A = -\frac{1}{2}x^2 + 6x$

$A' = -x + 6 = 0$

$6 = x$

$A_{max} = 18 u^2$

EXAMPLE 5

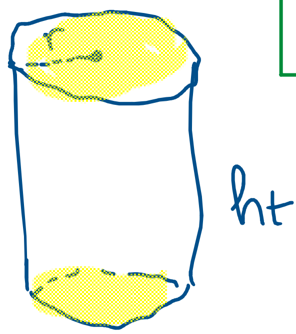
The owner of PEPSI wishes to make cans which hold 300 ml (300 cm^3) but he wishes to use as little metal as possible.

What should the radius be?

Hint: Volume of a cylinder = $\pi r^2 h$

Side circumference

$\boxed{\text{Area} = \pi d . ht}$ ht



$M = 2\pi r^2 + \pi d . ht$

$M = 2\pi r^2 + 2\pi r . ht$

kodwa $\pi r^2 . ht = V = 300$

$h = \dfrac{300}{\pi r^2}$

$M = 2\pi r^2 + 2\pi r . \left(\dfrac{300}{\pi r^2}\right)$

$M = 2\pi r^2 + 600 r^{-1}$

$M' = 4\pi r - 600 r^{-2} = 0$

$4\pi r = 600 r^{-2}$

$4\pi r = \dfrac{600}{r^2}$

$4\pi r^3 = 600$

$r^3 = \dfrac{600}{4\pi}$ $r = \sqrt[3]{\dfrac{600}{4\pi}}$