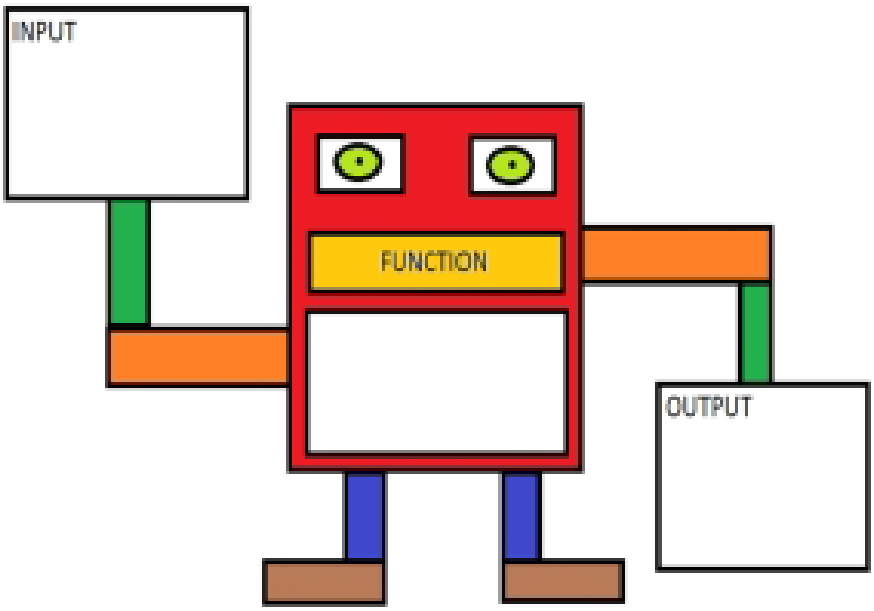
Grade 8 Mathematics

Module 7: Functions and Relationships



In this module we will look to identify relationships between inputs and outputs across a range of contexts. We will express these using the languages of both English and Maths (algebra)

EXERCISE 1

A taxi driver charges R40 to get to your home and then R4 per km from there.

a. What will a journey of 20 km cost?

b. What will a journey of 100 km cost?

c. What will a journey of *d* km cost?

d. What will a journey of *k* km cost?

e. If I spent R60 then how far did I travel?

f. If I spent R200 then how far did I travel?

g. If I spent *r* rands, then how far did I travel?

h. If I spent *p* rands, then how far did I travel?

EXERCISE 2

You and some friends each send one another a text.

Everyone must text everyone else.

See if you can complete the following table.

Try to find a rule or a pattern as you go.

You may want to draw some pictures initially…..

Number of people	Total number of texts
2	
3	
4	
5	
6	
20	
30	
40	

Can you complete the following?

If we have some people and they each send one another a text, then you can work out the total number of texts by taking the number of people and

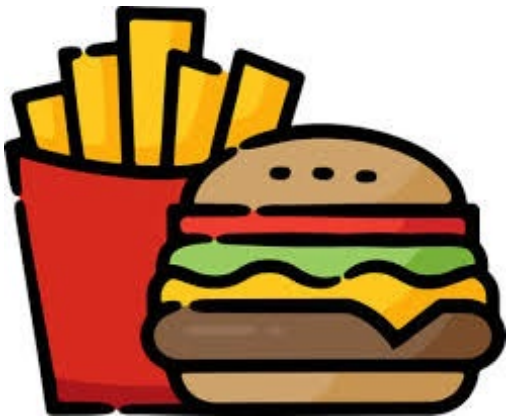
Can you express that using algebra?

If we have *n* people and they each send one another a text, then the total number of texts will be

EXERCISE 3

Complete the following table to indicate how much I have spent if I buy burgers at R20 each and chips at R10 per packet:

Number of burgers at R20 each	Number of packets of chips at R10 each	Total bill
1	2	
2	1	
	2	80
a	*b*	
3	5	
	d	*f*
g		*i*



EXERCISE 4

See if you can find the pattern in each of the following tables:

Hint: in this first one we are multiplying by a fixed number then adding a fixed number.
Both are positive numbers less than 4!

Input	2	4	6	10			a	
Output	8	14	20		56	104		b

Hint: in this second one we are multiplying by a fixed number and then subtracting a fixed number.
Both are positive numbers less than 5!

Input	2	5	7	8			a	
Output	7	19	27		63	83		b

In this last one we are squaring and then adding a fixed number.

Input	1	2	3	5			a	
Output	3	6	11		51	102		b

EXERCISE 5

You and some friends each call one another. Everyone must speak to everyone else on a phone call.

See if you can complete the following table. Try to find a rule or a pattern as you go.

Think about where you have seen something similar before. How is this different?

Can we use what we learnt last time?

You may want to draw some pictures initially…..

Number of people	Number of calls
2	
3	
4	
5	
30	
40	

Can you complete the following?

If we have some people and they each speak to each other person on the phone, then you can work out the total number of phone calls by taking the number of people and

Can you express that using algebra?

If we have *n* people and they each speak to each other person on the phone, then the total number of phone calls will be:

EXERCISE 6

Oily Olly drills oil wells. He charges R5000 to move his drilling rig to a site and then R30 per meter that he drills.

a. What will a 10m hole cost?

b. What will a 50m hole cost?

c. What will a hole that is *h* meters deep cost?

d. If I spent R8000 then how deep did he drill?

e. If I spent R5150 then how deep did he drill?

f. If I spent *k* rands then how many meters did he drill?

EXERCISE 1

Think of a number!

Carry out the following steps:

English	**Example**			
Think of a number				
Add 3				
Double the result				
Add 4				
Halve your answer				
Subtract the number you started with				

Compare your answers. What do you notice? Do you think this will ALWAYS be true?

Now try this one!

English	**Example**			
Think of a number				
Subtract 2				
Double your answer				
Add 8				
Halve your answer				
Subtract 2				

Compare your answers. What do you notice? Do you think this will ALWAYS be true?

It seems as if the first one always ends in 5 while the second always ends with the number you started with. Weird?! Not really, ***ALGEBRA*** is a mathematical language which can help us explain why!

ALGEBRA is powerful as it enables us to generalise situations. Rather than using a specific number we can use a ***variable*** – a ***variable*** is a letter whose value can ***vary***.

Let's do those exercises again using ***ALGEBRA***!

English	**Algebra**
Think of a number	
Add 3	
Double the result	
Add 4	
Halve your answer	
Subtract the number you started with	

English	**Algebra**
Think of a number	
Subtract 2	
Double your answer	
Add 8	
Halve your answer	
Subtract 2	

Now we are going to show you how to make up two *"think of a number"* problems.

Hint: it is easiest to start with the algebra then fill in the English and finally try an example.

This one is going to always end in 3!

English	Algebra	Example

This one is going to always end in the number we started with!

English	Algebra	Example

EXERCISE 2

Make up a **THINK OF A NUMBER PROBLEM** which always ends in 7

English	Algebra	Example

EXERCISE 3

Make up a **THINK OF A NUMBER PROBLEM** which always comes back to the number you started with.

English	Algebra	Example

EXERCISE 4

Converting English to Maths!

The power of algebra lies in our ability to take “real world” situations and express them mathematically.

Let’s try some examples:

a. In a class there are *p* boys and there are 5 more girls than boys. How many girls are there?

b. Give a number that is 3 less than *x*.

c. A hamburger costs twice as much as a coke. If a coke costs *c* then how much will a hamburger cost?

d. If there are *t* trout in a dam and *b* bass then how many fish are there altogether?

e. If a packet of chips costs *p* cents and I want to buy *n* packets then how much will I spend?

f. A coke costs twice as much as a chocolate. If the coke costs *g* then what does the chocolate cost?

g. A boy is five years older than his sister. If he is *j* years old then how old is his sister?

h. There are 3 tennis balls in a tin. If I have *t* tins then how many tennis balls do I have?

i. I have *b* brothers and *s* sisters. Express algebraically if:

   i. The number of siblings I have

   ii. The number of children in my family

j. I am currently *m* years old and my friend is currently *f* years old.

   i. How old will I be in 5 years time?

   ii. How old will my friend be in 5 years time?

   iii. What will be the sum of our ages in 5 years time?

   iv. How old was I 3 years ago?

   v. Double my age in 5 years time

   vi. Triple my age 3 years ago

k. How many days are there in *n* weeks and *p* days?

Some TERMinology!

Algebraic expressions are made up of

Terms are separated by + and – signs.

However, signs within fractions or brackets do not separate terms!

For example:

$2x + y$	has 2 terms
$\frac{4 + y}{x}$	is just one term
$(3 + c) + d$	has two terms
$\sqrt{x-2} - 6y + (p + q)^2$	has three terms

EXERCISE 5

How many terms in each of the following expressions?

a. $2xy + 3x(2x + 1)$

b. $2x + \frac{2xy + 5y}{x + 3} + 7y$

c. $(t-5)(t+3) + \sqrt{t-7} + \frac{6t+7}{t-2}$

d. $4x^3$

e. $6(x-4)^3$

A only has one term e.g.

A has two terms e.g.

A has three terms e.g.

A has many terms – more than 1!

We normally write a polynomial in descending (biggest to smallest) powers.

EXERCISE 6

Write the following polynomials in standard form without changing their values

a. $-3 + 5x^2 - 4 + 7x$

b. $4 - 6x^2 + 7x^3 + 3x$

c. $6 - 3x^2 + 5x^4 - x^3 + 8x$

The ***degree*** of a polynomial is the

For example, the polynomial $4x^5 + 5x^2 - 8x + 2$ has ***degree***

The ***coefficient*** of a variable is the number preceding it with the accompanying sign.

For example, in the polynomial $4x^5 + 5x^2 - 8x + 2$ the

coefficient of x^2 is

while the ***coefficient*** of x is

The ***constant*** term is one which does not involve a variable. It is fixed / constant – it cannot vary like a variable! For example, in the polynomial $4x^5 + 5x^2 - 8x + 2$, the ***constant*** term is

EXERCISE 7

1. Consider the polynomial: $5x^5 - 4x^4 + 3x^3 - 2x^2 - 5x - 7$

   a. How many terms does it have?

   b. What is the degree of the polynomial?

   c. What is the coefficient of x^2?

   d. What is the constant term?

   e. Evaluate ("work out the value") of the polynomial if $x = 1$

2. Consider the following expression:

   $3x^2 + 5x - 2y + 8$

   a. How many terms does it have?

   b. What is the constant term?

   c. What is the coefficient of y?

3. Write the following expressions in alphabetical order **without changing their value**:

Remember that it is the sign in front of (to the left of!) a term that is important!

a. $-4b + 8e - a + 3c - 2f + 6d$

b. $3z - 5w + 2u - 8v + \sqrt{x} - 3y$

c. $-3c + 5b - 2a + 4e - f + 6d$

Substituting values into algebraic expressions

We need to learn to evaluate (find the value of) algebraic expressions when given the values of the variables in them.

This is an excellent opportunity to practise our integer work as well as our general number skills!

Let's start by doing some examples together:

If $x = 2$ and $y = 5$ then evaluate (determine the value of):

$2x + 3y$

$3x - 2y$

x^2y

xy^2

It can get a little more interesting if we substitute negative numbers:

If $x = -2$ and $y = -3$ then evaluate (determine the value of):

$2x + 3y$

$3x - 2y$

x^2y

xy^2

Simplifying algebraic expressions

First, a few thoughts on adding things!

Suppose we wish to add $4 and €5.

We cannot add them as they are **different types** of things!

However, we could:

- convert the dollars to euros and then add them OR
- convert the euros to dollars and then add them OR
- convert both to Rands (or another currency) and then add them.

Suppose we want to add one-fifth $\frac{1}{5}$ to two-thirds$\frac{2}{3}$

We cannot add them as they are **different types** of things!

However, we could:

- convert both to fifteenths (or another common denominator) and then add them.

Suppose we want to add 10 cm and 7 mm.

We cannot add them as they are **different types** of things!

However, we could:

- convert the cm to mm and then add them OR
- convert the mm to cm and then add them OR
- convert both to m (or another unit) and then add them.

Suppose we have 3 bananas and 2 oranges

We can't add these as they are **different types** of thing!
By the way, there's no such thing as an orange banana 😊

Unlike with the money, the fractions and the lengths, there's no way of:

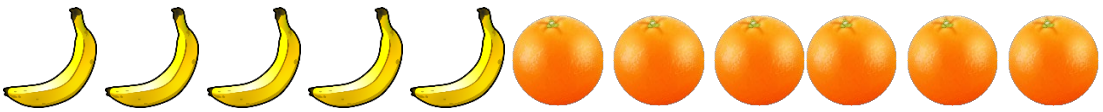
- converting a banana to an orange OR
- an orange to a banana OR
- both to a mango!

So, what do we do?

Well, we keep them separate:

If someone gives us another 2 bananas and another 4 oranges: then we have this:

We can collect the **same types** together like this:



And we say that we have five bananas and six oranges.

Something very similar happens with algebra.

Since **different variables (letters) represent different numbers**, we cannot simply add them.

Let's consider a couple of examples:

Add $5x$ and $7y$

Add $5x + 3$ and $7y + 6$

Add $5x + 2y$ and $7y + 6x$

EXERCISE 10

Now let's simplify the following expressions:

a. $3a + 2b + 7 + 8a - 6b + 12$

b. $3ab + 5 + 6 + 6ba + a + 2$ $3ab + 5 + b + 6ba + a + 2$

c. $2x^2y + 5xy^2 + 7yx^2 + 3y^2x$ $2x^2y + 5xy^2 + 7yx^2 + 3y^2x$

Let's summarise what we have learnt when simplifying algebraic expressions:

Remember!

- Order of operations – ebodmas!
- **NEVER, NEVER, NEVER** add unlike terms.

  For example, $x + y$ stays as $x + y$
- xy means $x \times y$
- Put the factors in alphabetical order with the coefficient (the number) at the front!

  e.g. $2ab$ rather than $2ba$ or $ab2$ even though all 3 are the same!
- Put an equals sign on every line.

  Remember that you are not altering the value – merely writing it in a different form.

EXERCISE 11

1. Simplify the following expressions:

   a. $x + x + x + x$

   b. $p \times p \times p \times p$

   c. $2 + x + 5 + 3x$

   d. $a + a + b + b$

   e. $2 \times a + 4 \times b$

   f. $3x + 5y - 7x + 2y$

g. $3 \times b \times a$

h. $y + y + y - x - x$

i. $4 \times x + (3 + 2) \times x$

j. $t \times t \times t \times t \times t \times t$

k. $p + 3 + q + 7$

l. $2x + 4 + y + 3x - 9 - 4y$

m. $t + w + 5t - (-3w)$

n. $t + t + t + t \times t \times t$

o. $2 + (-x) - (+5) - (-3x)$

p. $3x \times 4x$

q. $3xy \times 5y$

r. $2ab \times 4a$

s. $u \times u \times u \times u + u + u + u$

EXERCISE 12

Simplify the following:

a. $2z-(-6y)+(-7z)-(+8y)$

b. $e+e+e+f+e+f$

c. $4c+2-6c+3+d$

d. $f \times f \times f \times f \times f$

e. $5a^2b-3b^2a+4ab^2-2ba^2$

f. $g+g+g \times g \times g+g$

g. $3ab+4b-5ba-3a$

h. $4+2xy+5+3yx+x$

More practice at simplifying expressions!

We are going to go a little deeper into multiplication and then include that skill in some more examples involving addition.

Suppose we have

$$4x^4y^2 \times 5x^3y$$

Let's practise a few of those:

EXERCISE 13

Simplify the following:

$-2x^3y^5 \times -5x^4y^2$

$3a^4b^5 \times 6ab^4c^2$

$(a^2b^5)^2$

$(x^5y^2)^3$

When we multiply numbers with the same we their .

For example: $x^5y^4z^8 \times x^2y^3z =$

When we raise a number with a power to a power then we their

For example: $(x^4)^5 =$

EXERCISE 14

Simplify the following:

1. $3mn - 2m \times 4n$

2. $2x \times 3y \times x^2$

3. $3x^2a - 5a^2x + 2ax^2 - 3xa^2$

4. $4ab + 2a \times 3b$

5. $4x^2y \times 3x^5y^3$

6. $2ab \times 5b + (-3a)(2b^2)$

7. $(a^2b)^3 \times a^4b^5$

8. $2x - 5 + (-3x) - (-8)$

9. $2pq^2 + 5qp^2 - 7q^2p + 8p^2q$

10. $2ab + 5a \times -3b$

11. $ab \times 2b + 3b^2 \times 5a$

12. $(-2)(p)(-3q) + 2p \times 5q$