

List of Cubics with Rational Roots and Rational Turning Points

(* Indicates cubics without repeated roots)

(I've tried to weed out repetitions manually but the eyes go funny after a while so excuse the missed ones!)

*f(x) = (x-9)(x+6)(x-2) = $x^3-5x^2-48x+108$	f(x) = $3x^2-10x-48$
*f(x) = (x-9)(2x-3)(x+3) = $2x^3-15x^2-36x+81$	f(x) = $6x^2-30x-36$
*f(x) = (x+9)(x+5)(2x+3) = $2x^3+31x^2+132x+135$	f(x) = $6x^2+62x+132$
*f(x) = (x+9)(x-3)(2x+3) = $2x^3+15x^2-36x-81$	f(x) = $6x^2+30x-36$
*f(x) = (2x-9)(x-3)(2x-1) = $4x^3-32x^2+69x-27$	f(x) = $12x^2-64x+69$
*f(x) = (2x-9)(x+3)(2x-1) = $4x^3-8x^2-51x+27$	f(x) = $12x^2-16x-51$
*f(x) = (2x-9)(x+6)(x-2) = $2x^3-1x^2-60x+108$	f(x) = $6x^2-2x-60$
*f(x) = (2x-9)(2x-1)(x-2) = $4x^3-28x^2+49x-18$	f(x) = $12x^2-56x+49$
*f(x) = (2x+9)(x-3)(2x+1) = $4x^3+8x^2-51x-27$	f(x) = $12x^2+16x-51$
*f(x) = (2x+9)(x+2)(2x+1) = $4x^3+28x^2+49x+18$	f(x) = $12x^2+56x+49$
*f(x) = (2x+9)(x+3)(2x+1) = $4x^3+32x^2+69x+27$	f(x) = $12x^2+64x+69$
*f(x) = (2x+9)(x+6)(x+2) = $2x^3+25x^2+96x+108$	f(x) = $6x^2+50x+96$
*f(x) = (2x+9)(2x+1)(x+3) = $4x^3+32x^2+69x+27$	f(x) = $12x^2+64x+69$
*f(x) = (2x+9)(2x+1)(x+2) = $4x^3+28x^2+49x+18$	f(x) = $12x^2+56x+49$
*f(x) = (x-7)(x-3)(2x+1) = $2x^3-19x^2+32x+21$	f(x) = $6x^2-38x+32$
*f(x) = (x+7)(x+3)(2x-1) = $2x^3-3x^2-72x-35$	f(x) = $6x^2+38x+32$
*f(x) = (x-7)(x+5)(2x+1) = $2x^3-3x^2-72x-35$	f(x) = $6x^2-6x-72$
*f(x) = (x+7)(2x+7)(2x-1) = $4x^3+40x^2+77x-49$	f(x) = $12x^2+80x+77$
*f(x) = (x+7)(2x+9)(x+3) = $2x^3+29x^2+132x+189$	f(x) = $6x^2+58x+132$
*f(x) = (x-6)(x-3)(x+2) = x^3-7x^2+36	f(x) = $3x^2-14x$
*f(x) = (x+6)(x-2)(x+3) = x^3+7x^2-36	f(x) = $3x^2+14x$
*f(x) = (x-6)(x-2)(2x+3) = $2x^3-13x^2+36$	f(x) = $6x^2-26x$
*f(x) = (x+6)(2x-3)(x+2) = $2x^3+13x^2-36$	f(x) = $6x^2+26x$
*f(x) = (x-5)(x-2)(x+3) = $x^3-4x^2-11x+30$	f(x) = $6x^2-2x-11$
*f(x) = (2x-5)(x+5)(2x+3) = $4x^3+16x^2-35x-75$	f(x) = $12x^2+32x-35$
*f(x) = (2x-7)(x-2)(2x+1) = $4x^3-20x^2+17x+14$	f(x) = $12x^2-40x+17$
*f(x) = (2x-7)(x+4)(2x+1) = $4x^3+4x^2-55x-28$	f(x) = $12x^2+8x-55$
*f(x) = (2x+7)(x+2)(2x-1) = $4x^3+20x^2+17x-14$	f(x) = $12x^2+40x+17$
*f(x) = (2x+7)(x+6)(x+2) = $2x^3+23x^2+80x+84$	f(x) = $6x^2+46x+80$
*f(x) = (x-3)(x+5)(x+2) = $x^3+4x^2-11x-30$	f(x) = $3x^2+8x-11$
*f(x) = (x-2)(x+2)(2x-1) = $2x^3-x^2-8x+4$	f(x) = $6x^2-2x-8$
*f(x) = (x-2)(x+2)(2x+1) = $2x^3+x^2-8x-4$	f(x) = $6x^2+2x-8$
*f(x) = (x-2)(x+3)(3x+1) = $3x^4+4x^2-17x-6$	f(x) = $9x^2+8x-17$
*f(x) = (x-2)(x+3)(3x+2) = $3x^3+5x^2-16x-12$	f(x) = $9x^2+10x-16$
*f(x) = (x-2)(2x+3)(3x+2) = $6x^3+x^2-20x-12$	f(x) = $18x^2+2x-20$
*f(x) = (x-2)(3x-2)(2x+1) = $6x^3-13x^2+4$	f(x) = $18x^2-26x$
*f(x) = (x-2)(3x-1)(3x+2) = $9x^3-15x^2-8x+4$	f(x) = $27x^2-30x-8$
*f(x) = (x-1)(x-2)(3x+2) = $3x^3-7x^2+4$	f(x) = $9x^2-14x$
*f(x) = (x-1)(x+3)(2x+1) = $2x^3+5x^2-4x-3$	f(x) = $6x^2+10x-4$
*f(x) = (x-1)(x+3)(2x+3) = $2x^3+7x^2-9$	f(x) = $6x^2+14x$
*f(x) = (x-1)(x+7)(x+2) = $x^3+8x^2+5x-14$	f(x) = $3x^2+16x+5$
*f(x) = (x-1)(2x-1)(3x+1) = $6x^3-7x^2+1$	f(x) = $18x^2-14x$
*f(x) = (x-1)(2x+1)(x+3) = $2x^3+5x^2-4x-3$	f(x) = $6x^2+10x-4$
*f(x) = (x-1)(2x+3)(3x+1) = $6x^3+5x^2-8x-3$	f(x) = $18x^2+10x-8$
*f(x) = (x+1)(x+2)(3x-2) = $3x^3+7x^2-4$	f(x) = $9x^2+14x$
*f(x) = (x+1)(2x+1)(3x-1) = $6x^3+7x^2-1$	f(x) = $18x^2+14x$
*f(x) = (x+1)(x+6)(x-2) = $x^3+5x^2-8x-12$	f(x) = $3x^2+10x-8$
*f(x) = (x+1)(2x+7)(2x-1) = $4x^3+16x^2+5x-7$	f(x) = $12x^2+32x+5$
*f(x) = (x+2)(x+3)(3x+1) = $3x^3+16x^2+23x+6$	f(x) = $9x^2+32x+23$
*f(x) = (x+2)(2x-1)(3x+2) = $6x^3+13x^2-4$	f(x) = $18x^2+26x$
*f(x) = (x+2)(2x+3)(3x+2) = $6x^3+25x^2+32x+12$	f(x) = $18x^2+50x+32$
*f(x) = (x+2)(3x-2)(3x+1) = $9x^3+15x^2-8x-4$	f(x) = $27x^2+30x-8$
*f(x) = (x+2)(3x+1)(3x-2) = $9x^3+15x^2-4x-3$	f(x) = $18x^2+34x-4$
*f(x) = (x+3)(3x-2)(3x-2) = $9x^3+15x^2-32x+12$	f(x) = $27x^2+30x-32$
*f(x) = (2x-9)(2x+7)(2x+1) = $8x^3-4x^2-130x-63$	f(x) = $24x^2-8x-130$
f(x) = (x-2)(x-2)(x+2) = x^3-2x^2-4x+8	f(x) = $3x^2-4x-4$
f(x) = (x-2)(x-2)(x+3) = $x^3-x^2-8x+12$	f(x) = $3x^2-2x-8$
f(x) = (x-2)(x-2)(2x-1) = $2x^3-9x^2+12x-4$	f(x) = $6x^2-18x+12$

$f(x) = (x-2)(x-2)(2x+1) = 2x^3-7x^2+4x+4$
 $f(x) = (x-2)(x-2)(2x+3) = 2x^3-5x^2-4x+12$
 $f(x) = (x-2)(x-2)(3x-2) = 3x^3-14x^2+20x-8$
 $f(x) = (x-2)(x-2)(3x-1) = 3x^3-13x^2+16x-4$
 $f(x) = (x-2)(x-2)(3x+1) = 3x^3-11x^2+8x+4$
 $f(x) = (x-2)(x-2)(3x+2) = 3x^3-10x^2+4x+8$
 $f(x) = (x-2)(x+2)(x-2) = x^3-2x^2-4x+8$
 $f(x) = (x-2)(x+2)(x+2) = x^3+2x^2-4x-8$
 $f(x) = (x-2)(x+3)(x-2) = x^3-x^2-8x+12$
 $f(x) = (x-2)(x+3)(x+3) = x^3+4x^2-3x-18$
 $f(x) = (x-2)(2x-1)(x-2) = 2x^3-9x^2+12x-4$
 $f(x) = (x-2)(2x-1)(2x-1) = 4x^3-12x^2+9x-2$
 $f(x) = (x-2)(2x+1)(x+2) = 2x^3+x^2-8x-4$
 $f(x) = (x-2)(2x+1)(2x+1) = 4x^3+4x^2-7x-2$
 $f(x) = (x-2)(2x+3)(2x+3) = 4x^3+4x^2-15x-18$
 $f(x) = (x-2)(3x-2)(3x-2) = 9x^3-30x^2+28x-8$
 $f(x) = (x-2)(3x-1)(3x-1) = 9x^3-24x^2+13x-2$
 $f(x) = (x-2)(3x+1)(x+3) = 3x^3+4x^2-17x-6$
 $f(x) = (x-2)(3x+1)(3x+1) = 9x^3-12x^2-11x-2$
 $f(x) = (x-2)(3x+2)(3x+2) = 9x^3-6x^2-20x-8$
 $f(x) = (x-1)(x-2)(x-2) = x^3-5x^2+8x-4$
 $f(x) = (x-1)(x+2)(x+2) = x^3+3x^2-4$
 $f(x) = (x-1)(x+3)(x+3) = x^3+5x^2+3x-9$
 $f(x) = (x-1)(2x-1)(2x-1) = 4x^3-8x^2+5x-1$
 $f(x) = (x-1)(2x+1)(2x+1) = 4x^3-3x-1$
 $f(x) = (x-1)(2x+3)(2x+3) = 4x^3+8x^2-3x-9$
 $f(x) = (x-1)(3x-1)(3x-1) = 9x^3-15x^2+7x-1$
 $f(x) = (x-1)(3x-2)(3x-2) = 9x^3-21x^2+16x-4$
 $f(x) = (x-1)(3x+1)(3x+1) = 9x^3-3x^2-5x-1$
 $f(x) = (x-1)(3x+2)(3x+2) = 9x^3+3x^2-8x-4$
 $f(x) = (x+1)(x-2)(x-2) = x^3-3x^2+4$
 $f(x) = (x+1)(x+2)(x+2) = x^3+5x^2+8x+4$
 $f(x) = (x+1)(x+3)(x+3) = x^3+7x^2+15x+9$
 $f(x) = (x+1)(2x-1)(2x-1) = 4x^3-3x+1$
 $f(x) = (x+1)(2x+1)(2x+1) = 4x^3+8x^2+5x+1$
 $f(x) = (x+1)(2x+3)(2x+3) = 4x^3+16x^2+21x+9$
 $f(x) = (x+1)(3x-2)(3x-2) = 9x^3-3x^2-8x+4$
 $f(x) = (x+1)(3x-1)(3x-1) = 9x^3+3x^2-5x+1$
 $f(x) = (x+1)(3x+1)(3x+1) = 9x^3+15x^2+7x+1$
 $f(x) = (x+1)(3x+2)(3x+2) = 9x^3+21x^2+16x+4$
 $f(x) = (x+2)(x+2)(x+3) = x^3+7x^2+16x+12$
 $f(x) = (x+2)(x+2)(2x-1) = 2x^3+7x^2+4x-4$
 $f(x) = (x+2)(x+2)(2x+1) = 2x^3+9x^2+12x+4$
 $f(x) = (x+2)(x+2)(2x+3) = 2x^3+11x^2+20x+12$
 $f(x) = (x+2)(x+2)(3x-2) = 3x^3+10x^2+4x-8$
 $f(x) = (x+2)(x+2)(3x-1) = 3x^3+11x^2+8x-4$
 $f(x) = (x+2)(x+2)(3x+1) = 3x^3+13x^2+16x+4$
 $f(x) = (x+2)(x+2)(3x+2) = 3x^3+14x^2+20x+8$
 $f(x) = (x+2)(x+3)(x+2) = x^3+7x^2+16x+12$
 $f(x) = (x+2)(x+3)(x+3) = x^3+8x^2+21x+18$
 $f(x) = (x+2)(2x-1)(2x-1) = 4x^3+4x^2-7x+2$
 $f(x) = (x+2)(2x+1)(x-2) = 2x^3+x^2-8x-4$
 $f(x) = (x+2)(2x+1)(2x+1) = 4x^3+12x^2+9x+2$
 $f(x) = (x+2)(2x+3)(2x+3) = 4x^3+20x^2+33x+18$
 $f(x) = (x+2)(3x-2)(3x-2) = 9x^3+6x^2-20x+8$
 $f(x) = (x+2)(3x-1)(3x-1) = 9x^3+12x^2-11x+2$
 $f(x) = (x+2)(3x+1)(3x+1) = 9x^3+24x^2+13x+2$
 $f(x) = (x+2)(3x+2)(3x+2) = 9x^3+30x^2+28x+8$
 $f(x) = (x+3)(x+3)(2x-1) = 2x^3+11x^2+12x-9$
 $f(x) = (x+3)(x+3)(2x+1) = 2x^3+13x^2+24x+9$
 $f(x) = (x+3)(x+3)(2x+3) = 2x^3+15x^2+36x+27$
 $f(x) = (x+3)(x+3)(3x-2) = 3x^3+16x^2+15x-18$
 $f(x) = (x+3)(x+3)(3x-1) = 3x^3+17x^2+21x-9$
 $f(x) = (x+3)(x+3)(3x+1) = 3x^3+19x^2+33x+9$

$f'(x) = 6x^2-14x+4$
 $f'(x) = 6x^2-10x-4$
 $f'(x) = 9x^2-28x+20$
 $f'(x) = 9x^2-26x+16$
 $f'(x) = 9x^2-22x+8$
 $f'(x) = 9x^2-20x+4$
 $f'(x) = 3x^2-4x-4$
 $f'(x) = 3x^2+4x-4$
 $f'(x) = 3x^2-2x-8$
 $f'(x) = 3x^2+8x-3$
 $f'(x) = 6x^2-18x+12$
 $f'(x) = 12x^2-24x+9$
 $f'(x) = 6x^2+2x-8$
 $f'(x) = 12x^2-8x-7$
 $f'(x) = 12x^2+8x-15$
 $f'(x) = 27x^2-60x+28$
 $f'(x) = 27x^2-48x+13$
 $f'(x) = 9x^2+8x-17$
 $f'(x) = 27x^2-24x-11$
 $f'(x) = 27x^2-12x-20$
 $f'(x) = 3x^2-10x+8$
 $f'(x) = 3x^2+6x$
 $f'(x) = 3x^2+10x+3$
 $f'(x) = 12x^2-16x+5$
 $f'(x) = 12x^2-3$
 $f'(x) = 12x^2+16x-3$
 $f'(x) = 27x^2-30x+7$
 $f'(x) = 27x^2-42x+16$
 $f'(x) = 27x^2-6x-5$
 $f'(x) = 27x^2+6x-8$
 $f'(x) = 3x^2-6x$
 $f'(x) = 3x^2+10x+8$
 $f'(x) = 3x^2+14x+15$
 $f'(x) = 12x^2-3$
 $f'(x) = 12x^2+16x+5$
 $f'(x) = 12x^2+32x+21$
 $f'(x) = 27x^2-6x-8$
 $f'(x) = 27x^2+6x-5$
 $f'(x) = 27x^2+30x+7$
 $f'(x) = 27x^2+42x+16$
 $f'(x) = 3x^2+14x+16$
 $f'(x) = 6x^2+14x+4$
 $f'(x) = 6x^2+18x+12$
 $f'(x) = 6x^2+22x+20$
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 $f'(x) = 9x^2+26x+16$
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 $f'(x) = 6x^2+2x-8$
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 $f'(x) = 12x^2+40x+33$
 $f'(x) = 27x^2+12x-20$
 $f'(x) = 27x^2+24x-11$
 $f'(x) = 27x^2+48x+13$
 $f'(x) = 27x^2+60x+28$
 $f'(x) = 6x^2+22x+12$
 $f'(x) = 6x^2+26x+24$
 $f'(x) = 6x^2+30x+36$
 $f'(x) = 9x^2+32x+15$
 $f'(x) = 9x^2+34x+21$
 $f'(x) = 9x^2+38x+33$

Short List of Cubics with Rational Roots and Rational Turning Points

(The TPs with decimal fractions are messy and time consuming to display e.g. 3.7 is $3^{19}/_{27}$)

$*f(x)=(x-2)(x+2)(2x+1) = 2x^3+x^2-8x-4$	$f'(x) = 6x^2+2x-8$	TP: (1;-9), (-4/3;3.7)
$*f(x)=(x-2)(x+2)(2x-1) = 2x^3-x^2-8x+4$	$f'(x) = 6x^2-2x-8$	TP: (4/3;-3.71), (-1;-9)
$*f(x)=(x-2)(x+3)(3x+1) = 3x^3+4x^2-17x-6$	$f'(x) = 9x^2+8x-17$	TP: (1;-16), (-17/9;20.16)
$*f(x)=(x-2)(x+3)(3x-2) = 3x^3+5x^2-16x-12$	$f'(x) = 9x^2+10x-16$	TP: (8/9;-20.17), (-2;16)
$*f(x)=(x-2)(2x+1)(3x+2) = 6x^3+13x^2+4$	$f'(x) = 18x^2-26x$	TP: (13/9;-5.05), (0;4)
$*f(x)=(x-2)(2x+3)(3x-2) = 6x^3+x^2-20x-12$	$f'(x) = 18x^2+2x-20$	TP: (1;-25), (-10/9;3.22)
$*f(x)=(x-2)(3x-1)(3x+2) = 9x^3+5x^2-8x+4$	$f'(x) = 27x^2-30x-8$	TP: (4/3;-12), (-2/9;4.93)
$*f(x)=(x-2)(3x+2)(x+3) = 3x^3+5x^2-16x-12$	$f'(x) = 9x^2+10x-16$	TP: (8/9;-20.17), (-2;16)
$*f(x)=(x-1)(x-2)(3x+2) = 3x^3-7x^2+4$	$f'(x) = 9x^2-14x$	TP: (14/9;-1.65), (0;4)
$*f(x)=(x-1)(x+3)(2x+1) = 2x^3+5x^2-4x-3$	$f'(x) = 6x^2+10x-4$	TP: (1/3;-3.71), (-2;-9)
$*f(x)=(x-1)(x+3)(2x+3) = 2x^3+7x^2-9$	$f'(x) = 6x^2+14x$	TP: (0;-9), (-7/3;3.7)
$*f(x)=(x-1)(2x-1)(3x+1) = 6x^3-7x^2+1$	$f'(x) = 18x^2-14x$	TP: (7/9;-0.42), (0;1)
$*f(x)=(x-1)(2x+3)(3x+1) = 6x^3+5x^2-8x-3$	$f'(x) = 18x^2+10x-8$	TP: (4/9;-5.05), (-1;4)
$*f(x)=(x+1)(x+2)(3x-2) = 3x^3+7x^2-4$	$f'(x) = 9x^2+14x$	TP: (0;-4), (-14/9;1.64)
$*f(x)=(x+1)(2x+1)(3x-1) = 6x^3+7x^2-1$	$f'(x) = 18x^2+14x$	TP: (0;-1), (-7/9;0.41)
$*f(x)=(x+2)(x+3)(3x+1) = 3x^3+16x^2+23x+6$	$f'(x) = 9x^2+32x+23$	TP: (-1;-4), (-23/9;1.64)
$*f(x)=(x+2)(2x-1)(3x+2) = 6x^3+13x^2-4$	$f'(x) = 18x^2+26x$	TP: (0;-4), (-13/9;5.04)
$*f(x)=(x+2)(2x+3)(3x+2) = 6x^3+25x^2+32x+12$	$f'(x) = 18x^2+50x+32$	TP: (-1;-1), (-16/9;0.41)
$*f(x)=(x+2)(3x-2)(3x+1) = 9x^3+15x^2-8x-4$	$f'(x) = 27x^2-30x-8$	TP: (2/9;-4.94), (-4/3;11.99)
$*f(x)=(x+3)(2x-1)(3x+1) = 6x^3+17x^2-4x-3$	$f'(x) = 18x^2+34x-4$	TP: (1/9;-3.23), (-2;-25)
$*f(x)=(2x+3)(x-2)(3x+2) = 6x^3+x^2-20x-12$	$f'(x) = 18x^2+2x-20$	TP: (1;-25), (-10/9;3.22)

