Grade 12 Mathematics

Paper 1 Revision Workshop

15th August 2026

S·I·M·I

SHAYA IZIBALO MATHS INSTITUTE

NAME: <u>WORKED SOLUTIONS</u>

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ALGEBRA – expressions, equations, inequalities and nature of roots

1. Simplify, giving answers with positive exponents and rational denominators where applicable.

   A calculator may **not** be used.

a. $\frac{(a^2b^3)^{-1} \times ab^4}{a^{-4}b}$

$= \frac{a^{-2}b^{-3} \times ab^4}{a^{-4}b}$

$= \frac{a^{-1}b}{a^{-4}b}$

$= a^3$

b. $\left(\frac{1}{2}\right)^{x-1}\left(\sqrt{2}\right)^{2x-1}$

$= (2^{-1})^{x-1}\left(2^{\frac{1}{2}}\right)^{2x-1}$

$= 2^{-x+1}2^{x-\frac{1}{2}}$

$= 2^{\frac{1}{2}}$ or $\sqrt{2}$

c. $\frac{9^x - 3^{2x+1}}{3^{2x-1}}$

$= \frac{3^{2x} - 3^{2x+1}}{3^{2x-1}}$

$= \frac{3^{2x}(1-3)}{3^{2x-1}}$

$= 3(-2)$

$= -6$

d. $\frac{12^{x+1} \times 18^x}{8^{x+1} \times 27^{x-1}}$

$= \frac{(2^2 3)^{x+1} \times (2 \times 3^2)^x}{(2^3)^{x+1} \times (3^3)^{x-1}}$

$= \frac{2^{2x+2} \times 3^{x+1} \times 2^x \times 3^{2x}}{2^{3x+3} \times 3^{3x-3}}$

$= \frac{2^{3x+2} \times 3^{3x+1}}{2^{3x+3} \times 3^{3x-3}}$

$= 3^4 \times 2^{-1}$

$= \frac{81}{2}$

2. Solve for x without using a calculator (other than to evaluate logs):

a. $8^{2x+3} = 32^{x+1}$

$\therefore (2^3)^{2x+3} = (2^5)^{x+1}$

$\therefore 2^{6x+9} = 2^{5x+5}$

$\therefore 6x + 9 = 5x + 5$

$\therefore x = -4$

b. $3^{x+2} - 3^x = 72$

$\therefore 3^x(3^2 - 1) = 72$

$\therefore 3^x(8) = 72$

$\therefore 3^x = 9$

$\therefore 3^x = 3^2$

$\therefore x = 2$

c. $ax^{\frac{p}{c}} + b = g$

$\therefore ax^{\frac{p}{c}} = g - b$

$\therefore x^{\frac{p}{c}} = \dfrac{g-b}{a}$

$\therefore x = \left(\dfrac{g-b}{a}\right)^{\frac{c}{p}}$

d. $\dfrac{a}{x} + b = c$

$\therefore a + bx = cx$

$\therefore bx - cx = -a$

$\therefore x(b-c) = -a$

$\therefore x = \dfrac{-a}{b-c}$ or $x = \dfrac{a}{c-b}$

e. $x^2 - x < 12$

$\therefore x^2 - x - 12 < 0$

$\therefore (x-4)(x+3) < 0$

concave up parabolas are
negative BETWEEN their roots

$\therefore -3 < x < 4$

f. $\log_2(x+6) = 3$

$\therefore x + 6 = 2^3$

$\therefore x = 2^3 - 6$

$\therefore x = 2$

g. $2x^2 - 3x - 5 = 0$ by completing the square

$\therefore 2x^2 - 3x = 5$

$\therefore x^2 - \dfrac{3}{2}x = \dfrac{5}{2}$

$\therefore \left(x - \dfrac{3}{4}\right)^2 = \dfrac{5}{2} + \dfrac{9}{16}$

$\therefore \left(x - \dfrac{3}{4}\right)^2 = \dfrac{40}{16} + \dfrac{9}{16}$

$\therefore \left(x - \dfrac{3}{4}\right)^2 = \dfrac{49}{16}$

$\therefore x - \dfrac{3}{4} = \pm\dfrac{7}{4}$

$\therefore x = \dfrac{3}{4} \pm \dfrac{7}{4}$

$\therefore x = \dfrac{5}{2}$ or -1

h.

$\sqrt{x-2}+4=x$
$\therefore\sqrt{x-2}=x-4$
$\therefore x-2=x^2-8x+16$
$\therefore x^2-9x+18=0$
$\therefore(x-3)(x-6)=0$
$\therefore x=3 \text{ or } 6$
but a check reveals $x=6$ only

i.

$3(2^x)-17+10(2^{1-x})=0$
$\therefore 3(2^x)-17+10(2\times 2^{-x})=0$
$\therefore 3(2^x)-17+\frac{20}{2^x}=0$
$\therefore 3(2^x)^2-17(2^x)+20=0$
let $k=2^x$
$\therefore 3k^2-17k+20=0$
$\therefore(3k-5)(k-4)=0$
$\therefore k=\frac{5}{3} \text{ or } k=4$
$\therefore 2^x=\frac{5}{3} \text{ or } 2^x=4$
$\therefore x=\log_2\left(\frac{5}{3}\right) \text{ or } x=2$
$\therefore x=0{,}74 \text{ or } x=2$

j.

$2x^4+5x^3-x^2-6x=0$
$\therefore x(2x^3+5x^2-x-6)=0$
by inspection $x=1$ is a root of the cubic
so $(x-1)$ is a factor
$\therefore x(x-1)(2x^2+7x+6)=0$
$\therefore x(x-1)(2x+3)(x+2)=0$
$\therefore x=0 \text{ or } x=1 \text{ or } x=-\frac{3}{2} \text{ or } x=-2$

k.

$3x-2y=17$ and $4x+5y=-8$
multiplying first by 4
and second by 3 gives:
$12x-8y=68$
$12x+15y=-24$
subtracting these gives:
$-23y=92$
$y=-4$ and $x=3$
$\therefore(3;-4)$

3. Without solving discuss the nature of the roots of:

a. $2x^2 + 5x = 3$

$2x^2 + 5x - 3 = 0$

$\Delta = b^2 - 4ac = 25 - 4(2)(-3) = 49$

so, roots are rational and unequal

b. $x(x + 2) = 7$

$x^2 + 2x - 7 = 0$

$\therefore \Delta = b^2 - 4ac = 4 - 4(-7) = 32$

$\therefore$ roots are irrational and unequal

4. Find k if:

a. $x^2 + kx + 9 = 0$ has equal roots

$\Delta = 0$ when roots equal

$\therefore k^2 - 36 = 0$

$\therefore (k - 6)(k + 6) = 0$

$\therefore k = \pm 6$

b. $kx^2 + 2x + 5 = 0$ has real, unequal roots

$\Delta > 0$ when roots real and unequal

$\therefore 4 - 20k > 0$

$\therefore -20k > -4$

$\therefore k < \frac{1}{5}$

c. $x^2 + kx + \frac{k}{2} = -2$ has no real roots

$x^2 + kx + \frac{k}{2} + 2 = 0$

$\Delta < 0$ when no real roots

$\therefore k^2 - 4\left(\frac{k}{2} + 2\right) < 0$

$\therefore k^2 - 2k - 8 < 0$

$\therefore (k - 4)(k + 2) < 0$

concave up parabolas are
negative BETWEEN their roots

$\therefore -2 < k < 4$

5. Prove that the equation $x^2 + kx + k - 3 = 0$ has real roots for all values of k

we need to prove $\Delta > 0$

$x^2 + kx + k - 3 = 0$

$\Delta = k^2 - 4(k-3)$

$\Delta = k^2 - 4k + 12$

$\Delta = (k-2)^2 + 8$

which is clearly > 0

Sequences and Series

1. Determine, showing working:

a. T_{20} of 2 ; 6 ; 10 ; 14 ; ...

$T_n = a + (n-1)d$

$T_{20} = 2 + 19(4)$

$T_{20} = 78$

b. S_{30} of 3 ; $4\frac{1}{2}$; 6 ; $7\frac{1}{2}$; ....

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{30} = \frac{30}{2}\left[2(3) + 29\left(\frac{3}{2}\right)\right]$

$S_{30} = 742{,}5$

2. Determine k if $\sum_{i=4}^{23}(ki-2) = 1\,310$ showing all working details.

$\sum_{i=4}^{23}(ki-2) = (4k-2) + (5k-2) + (6k-2) + \ldots\text{to } 20 \text{ terms} = 1\,310$

$a = 4k - 2$ and $d = k$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = 10[2(4k-2) + 19k] = 1\,310$

$\therefore 10(27k - 4) = 1\,310$

$\therefore 27k - 4 = 131$

$\therefore 27k = 135$

$\therefore k = 5$

3. Which is the first term to exceed 20 000 000 in the sequence:

$6 ; 9 ; 13\frac{1}{2} ; 20\frac{1}{4} ; ....$

$6 ; 9 ; 13\frac{1}{2} ; 20\frac{1}{4} ; ....$

$r = \frac{3}{2}$

$T_n = ar^{n-1}$

$6\left(\frac{3}{2}\right)^{n-1} = 20\,000\,000$

$\left(\frac{3}{2}\right)^{n-1} = \frac{20\,000\,000}{6}$

$n - 1 = \log_{\frac{3}{2}}\left(\frac{20\,000\,000}{6}\right)$

$n = \log_{\frac{3}{2}}\left(\frac{20\,000\,000}{6}\right) + 1$

$n = 38{,}04260382$

so, T_{39} will be first to exceed 20 000 000

4. An arithmetic sequence has $T_9 = 13$ and $S_{10} = 60$. Find the first three terms.

$13 = a + 8d$ ① and $60 = \frac{10}{2}[2a + 9d]$ ②

from ① $a = 13 - 8d$

substituting into ② gives:

$60 = 5[2(13 - 8d) + 9d]$

$12 = 26 - 16d + 9d$

$-14 = -7d$

$d = 2$ and $a = -3$

so, first three terms are -3 ; -1 ; and 1

5. Which term in the sequence -1 ; 6 ; 17 ; 32 ; .... is equal to 899?

first differences: 7 ; 11 ; 15 ; ...

second differences: 4 ; 4 ; 4 ; ...

$a = 2$

$3(2) + b = 7$

so $b = 1$

$2 + 1 + c = -1$

$c = -4$

$T_n = 2n^2 + n - 4 = 899$

$\therefore 2n^2 + n - 903 = 0$

$\therefore n = 21$

6. Find p if:

a. $2p + 3$; $4p + 2$ and $5p + 3$ are consecutive terms of an arithmetic sequence.

$4p + 2 - (2p + 3) = 5p + 3 - (4p + 2)$
$\therefore 2p - 1 = p + 1$
$\therefore p = 2$

b. $3p + 1$; $2p + 4$ and $p + 8$ are consecutive terms of a geometric sequence.

$\dfrac{2p + 4}{3p + 1} = \dfrac{p + 8}{2p + 4}$
$\therefore (2p + 4)^2 = (3p + 1)(p + 8)$
$\therefore 4p^2 + 16p + 16 = 3p^2 + 25p + 8$
$\therefore p^2 - 9 + 8 = 0$
$\therefore (p - 1)(p - 8) = 0$
$\therefore p = 1$ or $p = 8$

c. $p - 1$; $2p - 1$; $4p - 2$ and $5p$ are consecutive terms of a quadratic progression.

$p - 1$; $2p - 1$; $4p - 2$ and $5p$
first differences are: $2p - 1 - (p - 1)$; $4p - 2 - (2p - 1)$; $5p - (4p - 2)$
first differences are: p ; $2p - 1$; $p + 2$
second differences are: $2p - 1 - p$ and $p + 2 - (2p - 1)$
these must be equal
$\therefore p - 1 = -p + 3$
$\therefore -4 = -2p$
$\therefore p = 2$

7. A radioactive substance weighing 8 kg loses 9% of its mass each week. After how many weeks will it first weigh less than a kilogram?

$8 \times 0{,}91 \ ; 8 \times 0{,}91^2 \ ; 8 \times 0{,}91^3 \ ; \ ...$

$T_n = ar^{n-1}$

$1 = 8 \times 0{,}91 \times 0{,}91^{n-1}$

$\dfrac{1}{8 \times 0{,}91} = 0{,}91^{n-1}$

$n - 1 = \log_{0{,}91}\left(\dfrac{1}{8 \times 0{,}91}\right)$

$n = \log_{0{,}91}\left(\dfrac{1}{8 \times 0{,}91}\right) + 1$

$n = 22{,}04884487$

so, after 23 weeks it will weigh less than 1 kg for the first time

8. A tree grows 2 m in its first year. Each year thereafter it grows by 20% of the amount it grew the year before. What is the maximum height it will ever reach?

The growth each year is given by the sequence:

$2 \ ; 2 \times 0{,}2 \ ; 2 \times 0{,}2^2 \ ; \ .....$

So, the total height is:

$2 \ + \ 2 \times 0{,}2 \ + \ 2 \times 0{,}2^2 \ ; \ .....$

this is a convergent geometric series since $r = 0{,}2$ and $-1 < r < 1$

$\therefore S_\infty = \dfrac{a}{1-r}$

$\therefore S_\infty = \dfrac{2}{1-0{,}2}$

$\therefore S_\infty = 2{,}5m$

9. For what value(s) of k will $\sum_{i=1}^{\infty} (2k+3)^i$ converge?

$$\sum_{i=1}^{\infty} (2k+3)^i = (2k+3) + (2k+3)^2 + (2k+3)^3 + \ .....$$

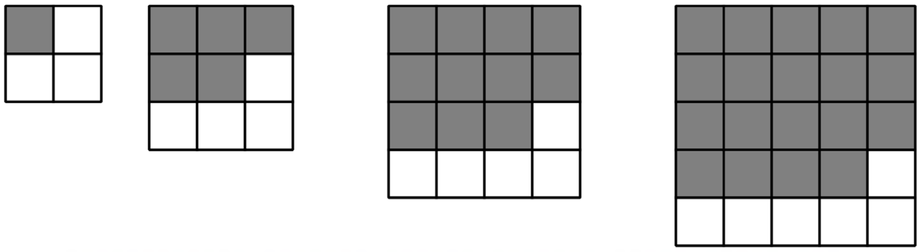
convergence occurs when $-1 < r < 1$

$\therefore -1 < 2k + 3 < 1$

$\therefore -4 < 2k < -2$

$\therefore -2 < k < -1$

10. How many shaded squares are there in figure 25?



the sequence is 1 ; 5 ; 11 ; 19 ; ...

first differences are 4 ; 6 ; 8 ; ....

second differences are 2 ; 2 ; 2 ; ....

so, $a = 1$

$3(1) + b = 4$ so $b = 1$

$1 + 1 + c = 1$ so $c = -1$

$T_n = n^2 + n - 1$

$T_{25} = 25^2 + 25 - 1$

$\underline{T_{25} = 649}$

However, if we see the structure we can get there quicker!
For shaded squares:

$T_n = (n+1)^2 - (n+1) - (1)$

$\therefore T_n = n^2 + 2n + 1 - n - 1 - 1$

$\therefore T_n = n^2 + n - 1$

$\therefore T_{25} = 25^2 + 25 - 1$

$\underline{\therefore T_{25} = 649}$

Financial Mathematics

1. Convert a rate of 10% p.a. compounded monthly to an effective annual rate. Express your answer as a percentage to 2 d.p. e.g., 10,15%

$$1\left(1+\frac{0,1}{12}\right)^{12}=1\left(1+\frac{i}{1}\right)^{1}$$

$$\therefore i=1\left(1+\frac{0,1}{12}\right)^{12}-1$$

$$\therefore i=10,47\%$$

2. What annual rate compounded quarterly will achieve the same return as 10% p.a.? Express your answer as a percentage to 2 d.p. e.g. 9,45%

$$1\left(1+\frac{i}{4}\right)^{4}=1\left(1+\frac{0,1}{1}\right)^{1}$$

$$\therefore 1+\frac{i}{4}=\left(1+\frac{0,1}{1}\right)^{\frac{1}{4}}$$

$$\therefore \frac{i}{4}=\left(1+\frac{0,1}{1}\right)^{\frac{1}{4}}-1$$

$$\therefore i=4\left[\left(1+\frac{0,1}{1}\right)^{\frac{1}{4}}-1\right]$$

$$\therefore i=9,65\%$$

3. If inflation is 6% p.a. then calculate, to the nearest year, how long it will take for an item to double in price.

$$A=P(1+i)^{n}$$

$$2x=x(1+0,06)^{n}$$

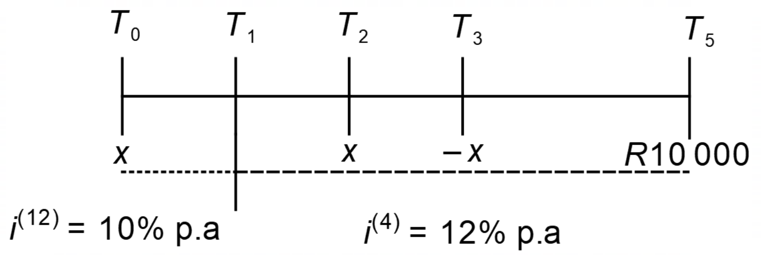
$$\therefore 2=(1+0,06)^{n}$$

$$\therefore n=\log_{1,06}(2)$$

$$\therefore n=11,9$$

so, 12 years

4. A man deposits x. Two years later he deposits a further x. A year later he withdraws x. Five years after his initial deposit he has R10 000. Find x to the nearest cent if interest was paid at 10% p.a. compounded monthly for the first year and then at 12% p.a. compounded quarterly thereafter.



Easiest to consider the amounts separately!

$$x\left(1+\frac{0,1}{12}\right)^{12}\left(1+\frac{0,12}{4}\right)^{16}+x\left(1+\frac{0,12}{4}\right)^{12}-x\left(1+\frac{0,12}{4}\right)^{8}=10\,000$$

$$\therefore x\left[\left(1+\frac{0,1}{12}\right)^{12}\left(1+\frac{0,12}{4}\right)^{16}+\left(1+\frac{0,12}{4}\right)^{12}-\left(1+\frac{0,12}{4}\right)^{8}\right]=10\,000$$

$$\therefore x=\frac{10\,000}{\left(1+\frac{0,1}{12}\right)^{12}\left(1+\frac{0,12}{4}\right)^{16}+\left(1+\frac{0,12}{4}\right)^{12}-\left(1+\frac{0,12}{4}\right)^{8}}$$

$$\therefore x = R5\,176,70$$

5. Dali deposits R300 per month into an account paying 10% p.a. compounded monthly. He makes 60 payments. How much money does he have in the account 4 months after making his last payment?

immediately after last payment he has:

$$F = x\left[\frac{(1+i)^n-1}{i}\right]$$

$$F = 300\left[\frac{\left(1+\frac{0,1}{12}\right)^{60}-1}{\frac{0,1}{12}}\right]$$

$$F = 23\,231,12$$

4 months later he has $23\,231,12\left(1+\frac{0,1}{12}\right)^{4}$

4 months after final payment he has 24 015,23

6. Jason borrows R200 000. He repays it monthly over 4 years starting 1 month from the date of the loan. Interest is charged at 12% p.a. compounded monthly. Calculate his monthly repayment.

$$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$200\,000 = x\left[\frac{1-\left(1+\frac{0,12}{12}\right)^{-48}}{\frac{0,12}{12}}\right]$$

$$\therefore x = \frac{200\,000}{\frac{1-\left(1+\frac{0,12}{12}\right)^{-48}}{\frac{0,12}{12}}}$$

$$\therefore x = R5\,266,77$$

7. Jordyn borrows R1 000 000 to buy a house. He repays it with monthly payments of R15 000 per month starting one month from the date of the loan. Interest is charged at 10% p.a. compounded monthly.

a. How long will it take him to repay the loan?

$$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$1\,000\,000 = 15\,000\left[\frac{1-\left(1+\frac{0,10}{12}\right)^{-n}}{\frac{0,1}{12}}\right]$$

$$\frac{1\,000\,000}{15\,000} = \frac{1-\left(1+\frac{0,10}{12}\right)^{-n}}{\frac{0,1}{12}}$$

$$\frac{1\,000\,000}{15\,000} \times \frac{0,1}{12} = 1-\left(1+\frac{0,10}{12}\right)^{-n}$$

$$\left(1+\frac{0,10}{12}\right)^{-n} = 1-\frac{1\,000\,000}{15\,000} \times \frac{0,1}{12}$$

$$\therefore -n = \log_{1+\frac{0,1}{12}}\left(1-\frac{1\,000\,000}{15\,000} \times \frac{0,1}{12}\right)$$

$$\therefore n = -\log_{1+\frac{0,1}{12}}\left(1-\frac{1\,000\,000}{15\,000} \times \frac{0,1}{12}\right)$$

$$\therefore n = 97,72$$

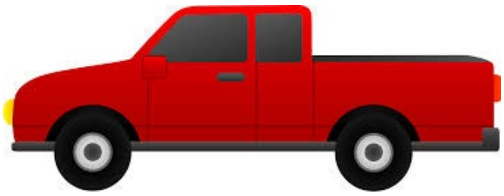
so, 98 months

b. How much does he still owe after his 30th payment?

$$\text{Outstanding balance} = 1\,000\,000\left(1+\frac{0,1}{12}\right)^{30} - 15\,000\left[\frac{\left(1+\frac{0,1}{12}\right)^{30}-1}{\frac{0,1}{12}}\right]$$

$$\underline{\text{Outstanding balance} = R773\,843,23}$$

8. George buys a new bakkie for his company. It costs R750 000. He plans to keep it for 5 years after which he will trade it in for its depreciated value. The bakkie will depreciate at 15% p.a. while inflation is expected to average 5,7% over the next 5 years. George plans to set up a sinking fund paying 9% p.a. compounded monthly. How much should he invest each month to be able to replace his bakkie at the end of the five-year period?



$$\begin{aligned}\text{Amount he will need in 5 years time} &= \text{price of new bakkie} - \text{depreciated value of old } one\\ &= 750\,000(1+0,057)^5 - 750\,000(1-0,15)^5\\ &= 656\,767,50\end{aligned}$$

$$F = x\left[\frac{(1+i)^n-1}{i}\right]$$

$$656\,767,50 = x\left[\frac{\left(1+\frac{0,09}{12}\right)^{60}-1}{\frac{0,09}{12}}\right]$$

$$\underline{x = R8\,707,66}$$

Calculus

1. Determine the following derivatives by first principles. Pay careful attention to notation:

a. $f(x) = -x^2 + 3x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{-(x+h)^2 + 3(x+h) - (-x^2 + 3x)}{h}$$

$$= \lim_{h \to 0} \frac{-x^2 - 2xh - h^2 + 3x + 3h + x^2 - 3x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}^1(-2x - h + 3)}{\cancel{h}^1}$$

$$= \lim_{h \to 0} (-2x - h + 3)$$

$$= -2x + 3$$

b. $f(x) = (x+1)^2$

$f(x) = x^2 + 2x + 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) + 1 - (x^2 + 2x + 1)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h + 1 - x^2 - 2x - 1}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}^1(2x + h + 2)}{\cancel{h}^1}$$

$$= \lim_{h \to 0} (2x + h + 2)$$

$$= 2x + 2$$

2. Determine the following derivatives paying careful attention to notation:

a. $y = \left(x - \frac{1}{x}\right)^2$

$= x^2 - 2 + \frac{1}{x^2}$

$= x^2 - 2 + x^{-2}$

$\therefore \frac{dy}{dx} = 2x - 2x^{-3} \quad or \quad 2x - \frac{2}{x^3}$

b. $f(x) = \frac{x^3 - 8}{2x - 4}$

$f(x) = \frac{(x-2)(x^2 + 2x + 4)}{2(x-2)}$

$f(x) = \frac{\cancel{(x-2)}^1 (x^2 + 2x + 4)}{2\cancel{(x-2)}^1}$

$f(x) = \frac{1}{2}x^2 + x + 2$

$f'(x) = x + 1$

'

c. $\frac{d}{dx}\left[\frac{5}{x^2} + \sqrt[3]{x^4} + \sqrt{5}\right]$

$= \frac{d}{dx}\left[5x^{-2} + x^{\frac{4}{3}} + \sqrt{5}\right]$

$= -5x^{-3} + \frac{4}{3}x^{\frac{1}{3}} \quad or \quad -\frac{5}{x^3} + \frac{4\sqrt[3]{x}}{3}$

d. $D_x\left[\frac{x^3 + 6x}{3x^2}\right]$

$= D_x\left[\frac{x}{3} + \frac{2}{x}\right]$

$= D_x\left[\frac{x}{3} + 2x^{-1}\right]$

$= \frac{1}{3} - 2x^{-2} \quad or \quad \frac{1}{3} - \frac{2}{x^2}$

3. Determine whether the given function is increasing, decreasing or stationary at the given value:

a. $f(x) = 2x^3 + x^2 - 8x - 4$ at $x = 1$

$f'(x) = 6x^2 + 2x - 8$

$f'(1) = 0$

so, f is stationary at $x = 1$

b. $f(x) = 2x^3 + 7x^2 - 9$ at $x = -1$

$f'(x) = 6x^2 + 14x$

$f'(-1) = -8$

so, f is decreasing at $x = -1$

4. For what value(s) of x is:

a. $y = x^3 + 4x^2 - 11x - 30$ increasing?

$\frac{dy}{dx} = 3x^2 + 8x - 11 > 0$

$(3x + 11)(x - 1) > 0$

concave up parabolas are positive outside of their roots

$\therefore x < -\frac{11}{3}$ or $x > 1$

b. $f(x) = 3x^3 + 16x^2 + 23x + 6$ concave down

$f'(x) = 9x^2 + 32x + 23$

$f''(x) = 18x + 32 < 0$

$\therefore 18x < -32$

$\therefore x < -\frac{32}{18}$

$\therefore x < -\frac{16}{9}$

5. Find the equation of the tangent to:

a. $y = x^2 + 5x - 3$ at the point where $x = 2$

the point of contact is $(2;11)$

$\frac{dy}{dx} = 2x + 5$

so, at $x = 2$, the gradient is 9.

$y - 11 = 9(x - 2)$

$y = 9x - 7$

b. $y = x^2 - 5$ which is perpendicular to the line $4y + x = 8$

the line is $4y = -x + 8$ or $y = -\frac{1}{4}x + 2$

which has a gradient of $-\frac{1}{4}$

perpendicular gradient = 4

$\frac{dy}{dx} = 2x = 4$

$\therefore x = 2$

so, the point of contact is $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$y + 1 = 4(x - 2)$

$y = 4x - 9$

6. Sketch $y = 6x^3 + 5x^2 - 8x - 3$ on the axes provided, showing all points of interest.

$y = 6x^3 + 5x^2 - 8x - 3$

y-intercept (0;-3)

x-intercepts: let $y = 0$

$6x^3 + 5x^2 - 8x - 3 = 0$

by inspection $x = 1$ is a root so $(x-1)$ is a factor

$(x-1)(6x^2 + 11x + 3) = 0$

$(x-1)(3x+1)(2x+3) = 0$

$x = 1 \text{ or } -\frac{1}{3} \text{ or } -\frac{3}{2}$

Stationary points: let $\frac{dy}{dx} = 0$

$18x^2 + 10x - 8 = 0$

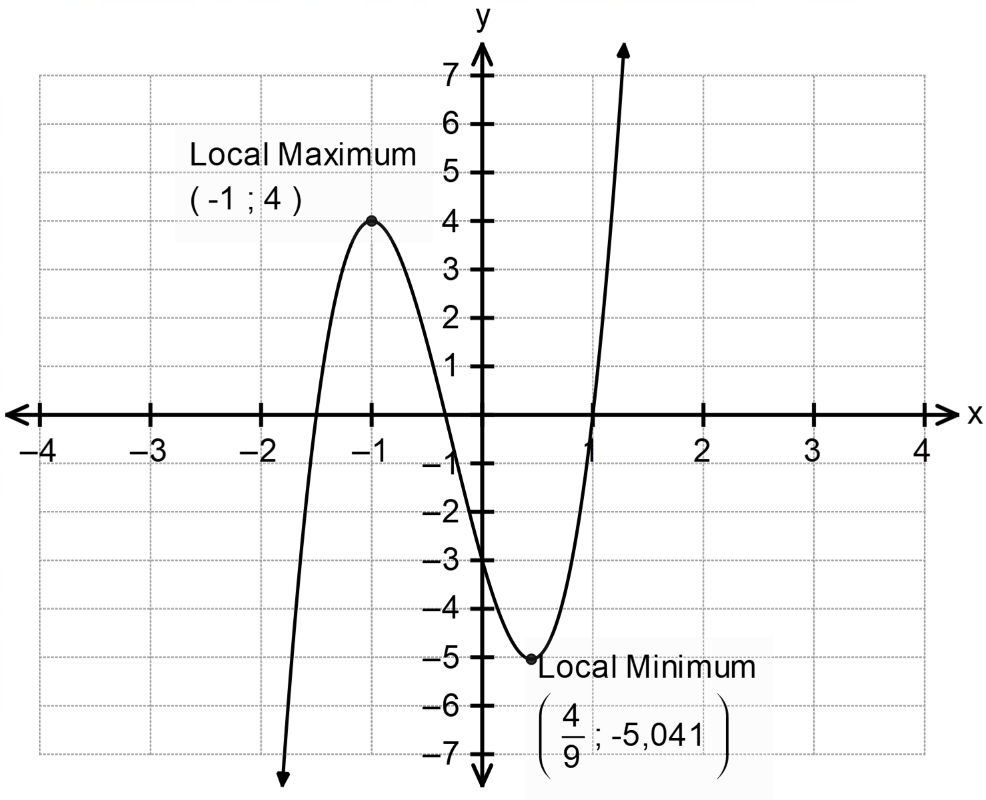
$9x^2 + 5x - 4 = 0$

$(9x-4)(x+1) = 0$

$x = \frac{4}{9} \text{ or } -1$

$\frac{d^2y}{dx^2} = 36x + 10$ is <0 at $x = -1$ so a maximum at (-1;4)

$\frac{d^2y}{dx^2} = 36x + 10$ is >0 at $x = \frac{4}{9}$ so a minimum at (-1;-5,04)



7. Find the point of inflection of $y = x^3 + 6x^2 - 8x + 13$

$y = x^3 + 6x^2 - 8x + 13$

$\frac{dy}{dx} = 3x^2 + 12x - 8$

$\frac{d^2y}{dx^2} = 6x + 12 = 0$

$x = -2$

(-2;45) is a point of inflection

8. The function $y = ax^2 + bx - 2$ has a gradient of 9 at the point (3;7). Find a and b.

since (3;7) lies on the function

$7 = 9a + 3b - 2$

$9 = 9a + 3b$ ①

$\frac{dy}{dx} = 2ax + b$

since the gradient is 9 when $x = 3$

$6a + b = 9$ ②

from ② $b = 9 - 6a$

substitute this into ①

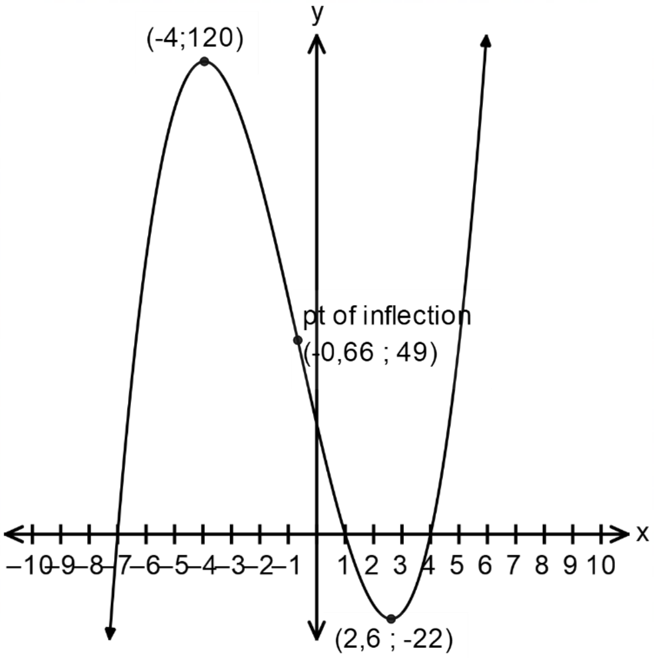
$9 = 9a + 3(9 - 6a)$

$9 = 9a + 27 - 18a$

$-18 = -9a$

$a = 2$ and $b = -3$

9. Consider the function $f(x)$ and answer the questions which follow:



Give all value(s) of *x* for which:

a. $f(x) < 0$

$x < -7$ or $1 < x < 4$

b. $f'(x) \geq 0$

$x \leq -4$ or $x \geq 2{,}6$

c. $f''(x) < 0$

$x < -0{,}66$

d. $f''(x) = 0$

$x = -0{,}66$

10. A fruit farmer has 300 apple trees each of which has 70 apples. Presently each apple can sell for 50c, but the price will increase by 10c per apple per week. Each week each tree loses 2 apples.

a. What is the crop presently worth?

$V = 300 \times 70 \times 50$
$V = 1\,050\,000c$ or $R10\,500$

b. What will the crop be worth in t weeks' time?

$V = 300 \times (70 - 2t) \times (50 + 10t)$
$V = 300 \times (-20t^2 + 600t + 3\,500)$
$V = -6\,000t^2 + 180\,000t + 1\,050\,000$

c. At what point should the farmer harvest his crop to maximise his profit?

$V = -6\,000t^2 + 180\,000t + 1\,050\,000$
$\frac{dV}{dt} = -12\,000t + 180\,000 = 0$
$-12\,000t + 180\,000 = 0$
$-12\,000t = -180\,000$
$t = 15$ weeks

d. What is the maximum profit?

$V = 2\,400\,000\ c$ or $R24\,000$

11. A boy has a 100 cm piece of rope. He cuts it into two pieces and forms a square and a circle.

a. Suppose the sides of the square are x. Find the area of both shapes in terms of x

Area of $square = x^2$

remaining rope for $circle = 100 - 4x$

circumference $= \pi \times 2r$

$$r = \frac{\text{circumference}}{2\pi} = \frac{100-4x}{2\pi}$$

$$\text{Area of circle} = \pi\left(\frac{100-4x}{2\pi}\right)^2$$

b. Find the value of x that will **minimize** the total area enclosed by the two shapes. Give your answer to the nearest cm.

$$A = x^2 + \pi\left(\frac{100-4x}{2\pi}\right)^2$$

$$A = x^2 + \pi\left(\frac{10\,000 - 800x + 16x^2}{4\pi^2}\right)$$

$$A = x^2 + \left(\frac{10\,000 - 800x + 16x^2}{4\pi}\right)$$

$$A = x^2 + \frac{2\,500}{\pi} - \frac{200x}{\pi} + \frac{4x^2}{\pi}$$

$$\frac{dA}{dx} = 2x - \frac{200}{\pi} + \frac{8x}{\pi} = 0$$

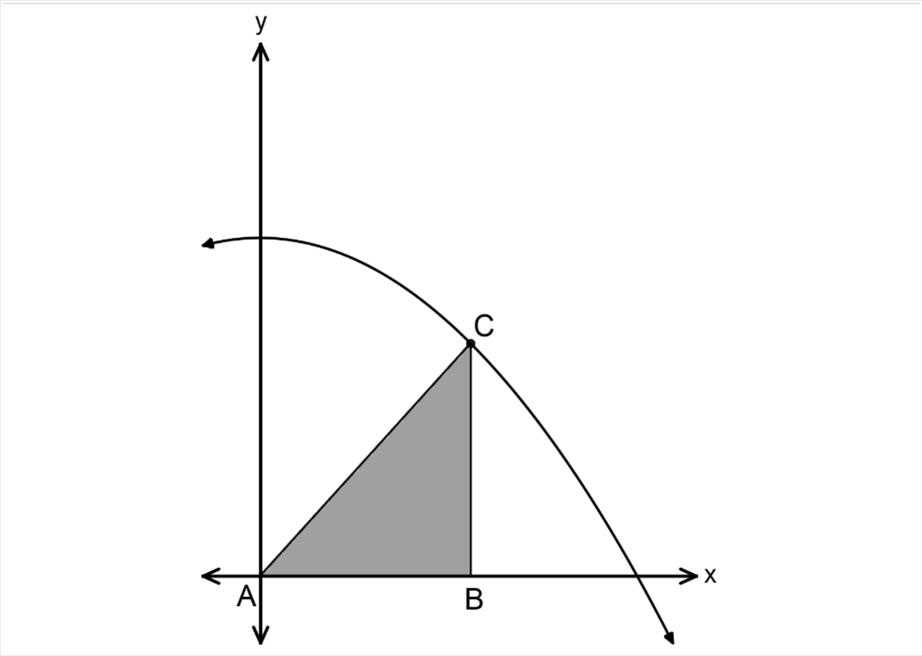
$$2\pi x - 200 + 8x = 0$$

$$x(2\pi + 8) = 200$$

$$x = \frac{200}{2\pi + 8}$$

$$x = 14$$

12. A right-angled triangle has one vertex on the origin and the other on the function $f(x) = -0,1x^2 + 8$ as shown.



a. If B has coordinates $(x;0)$ then give the coordinates of C in terms of x.

$C(x;-0,1x^2 + 8)$

b. Hence, or otherwise, find the area of ΔABC in terms of x.

$A = x(-0,1x^2 + 8)$

$A = -0,1x^3 + 8x$

c. Find the value of x which will give the maximum area.

$A = -0,1x^3 + 8x$

$\frac{dA}{dx} = -0,3x^2 + 8 = 0$

$-0,3x^2 = -8$

$x^2 = \frac{8}{0,3}$

$x = \sqrt{\frac{8}{0,3}} \approx 5,16\text{cm}$

d. Give the maximum area.

$A = 27,54u^2$

Functions and Inverses

1. For each of the following say, with justification, whether we have a function.

a. (-2;5) (1;3) (7;8) (1;5) (9;11)

no, x doesn't uniquely define y

b. $g(x) = \pm\sqrt{x+4}$

no, x doesn't uniquely define y

c. (-3;1) (2;5) (4;1) (5;6) (7;9)

yes, x uniquely defines y

d. $y = \sin x$

yes, x uniquely defines y

2. Give the domain and range of each of the following functions:

a. $y = \dfrac{12}{x-4} + 3$

Domain: $x \neq 4$

Range: $y \neq 3$

b. $y = \log_2(x+4)$

Domain: $x > -4$

Range: $y \in \mathbb{R}$

a. $y = 0{,}5^x - 9$

Domain: $x \in \mathbb{R}$

Range: $y > -9$

d. $y = -(x+4)^2 + 9$

Domain: $x \in \mathbb{R}$

Range: $y \leq 9$

3. Find the turning point and the range of the following functions by completing the square:

a. $y = x^2 + 6x + 11$

$= (x+3)^2 + 11 - 9$

$= (x+3)^2 + 2$

TP (-3;2)

range: $y \geq 2$

b. $y = x^2 + 5x - 6$

$= \left(x + \frac{5}{2}\right)^2 - 6 - 6\frac{1}{4}$

$= \left(x + \frac{5}{2}\right)^2 - 12\frac{1}{4}$

$TP\left(-\frac{5}{2}\,;\,-12\frac{1}{4}\right)$

range: $y \geq -12\frac{1}{4}$

4. Draw graphs of the following functions showing all points of interest. Where there are asymptotes, they should be drawn (with dotted lines) and labelled.

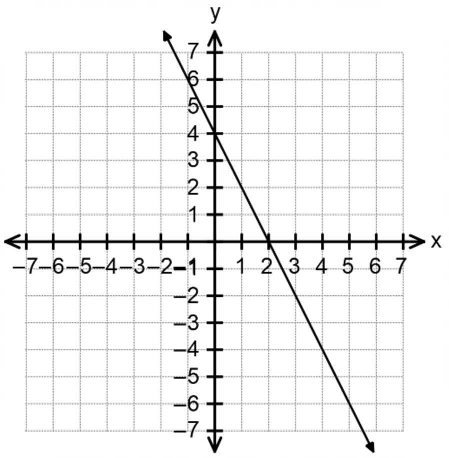
a. $y = -2x + 4$

y-intercept: (0;4)

x-intercept: $-2x + 4 = 0$

$4 = 2x$

(2;0)



b. $y = 0{,}5^x - 4$

horiz. asymp. $y = -4$

y-intercept: (0;-3)

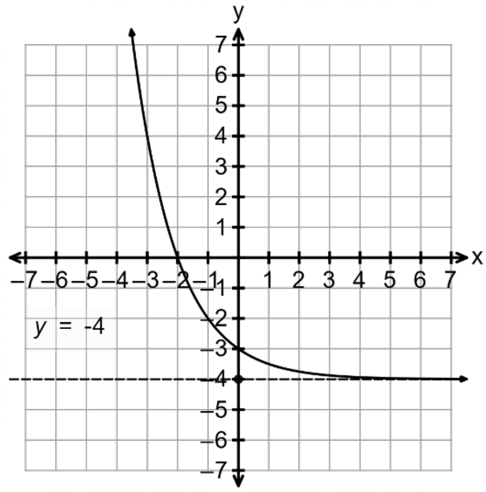
x-intercept: let $y = 0$

$0{,}5^x - 4 = 0$

$0{,}5^x = 4$

$2^{-x} = 2^2$

(-2;0)



c. $y = \log_2(x + 4)$

vert. asymp. $x = -4$

y-intercept: (0;2)

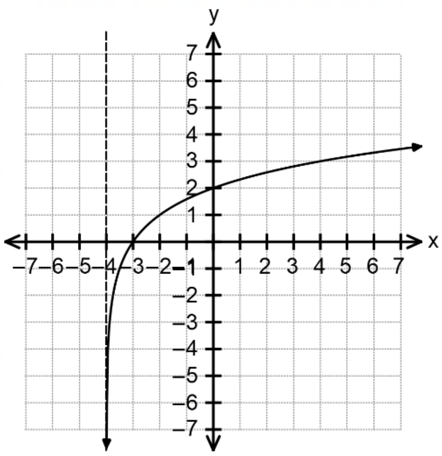
x-intercept: let $y = 0$

$0 = \log_2(x + 4)$

$2^0 = x + 4$

$1 = x + 4$

(-3;0)



d. $y = x^2 - 5x + 6$

y-intercept: (0;-6)

x-intercepts: let $y = 0$

$x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

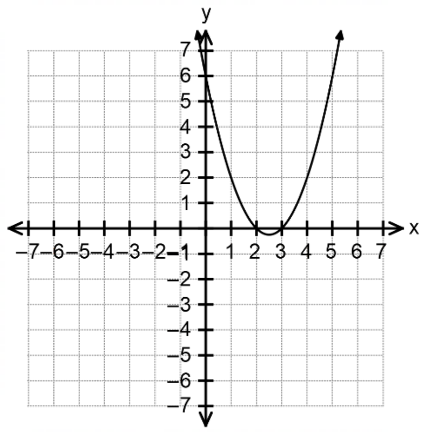
(3;0) and (2;0)

TP let $\frac{dy}{dx} = 0$

$2x - 5 = 0$

$x = \frac{5}{2}$

$\left(\frac{5}{2}; -\frac{1}{4}\right)$



e. $y = -2 \times \left(\frac{1}{3}\right)^x + 6$

hor. asymp. $y = 6$

y-intercept: (0;4)

x-intercept: let $y = 0$

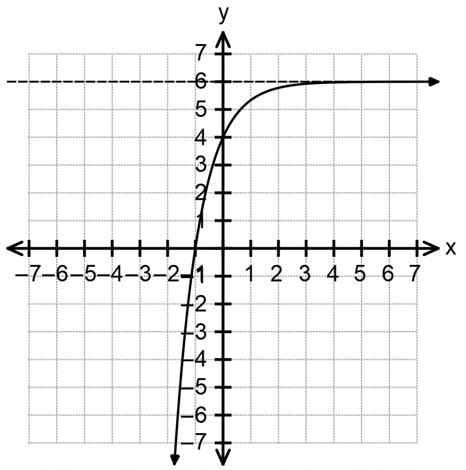
$$-2 \times \left(\frac{1}{3}\right)^x + 6 = 0$$

$$-2 \times \left(\frac{1}{3}\right)^x = -6$$

$$\left(\frac{1}{3}\right)^x = 3$$

$$3^{-x} = 3^1$$

(-1;0)



f. $y = \frac{-9}{x+4} - 2$

hor. asymp. $y = -2$

vert. aymp. $x = -4$

y-intercept $\left(0;-4\frac{1}{4}\right)$

x-intercept: let $y = 0$

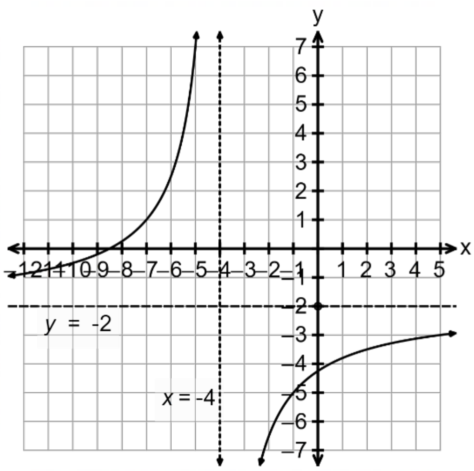
$$-\frac{9}{x+4} - 2 = 0$$

$$-\frac{9}{x+4} = 2$$

$$-9 = 2x + 8$$

$$-17 = 2x$$

$$\left(-8\frac{1}{2};0\right)$$



5. Give, in standard form, the equation of the function which results when:

a. $y = x^2 + 3x - 5$
is shifted 3 left and 2 up

$y = (x+3)^2 + 3(x+3) - 5 + 2$

$y = x^2 + 6x + 9 + 3x + 9 - 3$

$y = x^2 + 9x + 15$

b. $y = 3^x + 5$
is reflected in the x-axis

$y = -3^x - 5$

a. $y = \dfrac{7}{x+3} - 2$
is reflected in the y-axis

$y = \dfrac{7}{-x+3} - 2$

d. $y = 3^x - 5$
is reflected in the line $y = x$

we want the inverse:

$x = 3^y - 5$

$x + 5 = 3^y$

$y = \log_3(x+5)$

6. Find the inverses of the following functions in the form $f^{-1}(x) = \ldots$

a. $f(x) = 3x - 5$

$y = 3x - 5$

$x = 3y - 5$

$x + 5 = 3y$

$y = \dfrac{x+5}{3}$

$f^{-1}(x) = \dfrac{x+5}{3}$

b. $f(x) = 2 \times 3^x + 6$

$y = 2 \times 3^x + 6$

$x - 6 = 2 \times 3^y$

$\dfrac{x-6}{2} = 3^y$

$y = \log_3\left(\dfrac{x-6}{2}\right)$

$f^{-1}(x) = \log_3\left(\dfrac{x-6}{2}\right)$

c. $f(x) = -\dfrac{3}{x-4} + 5$

$y = -\dfrac{3}{x-4} + 5$

$x = -\dfrac{3}{y-4} + 5$

$x-5 = -\dfrac{3}{y-4}$

$(x-5)(y-4) = -3$

$y-4 = -\dfrac{3}{x-5}$

$y = -\dfrac{3}{x-5} + 4$

$f^{-1}(x) = -\dfrac{3}{x-5} + 4$

d. $f(x) = \log_2(x+3) - 5$

$y = \log_2(x+3) - 5$

$x = \log_2(y+3) - 5$

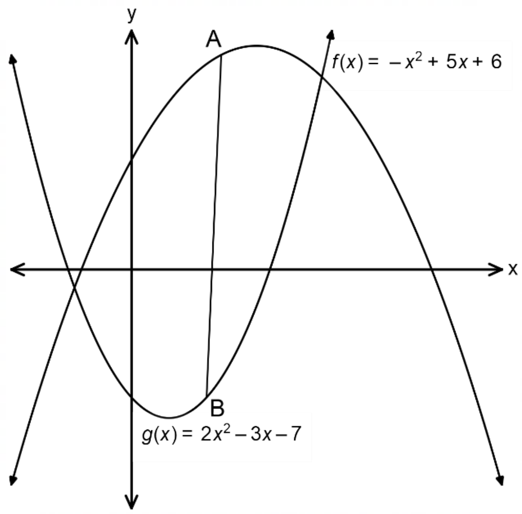
$x + 5 = \log_2(y+3)$

$y + 3 = 2^{x+5}$

$y = 2^{x+5} - 3$

$f^{-1}(x) = 2^{x+5} - 3$

7. Consider the sketch below. AB is a vertical line with A on *g* and B on *f*.
Find the maximum length of AB if it is on the interval where *g* exceeds *f*.



$AB =$ difference in y-values

$= -x^2 + 5x + 6 - (2x^2 - 3x - 7)$

$= -3x^2 + 8x + 13$

we wish to maximize this

$l = -3x^2 + 8x + 13$

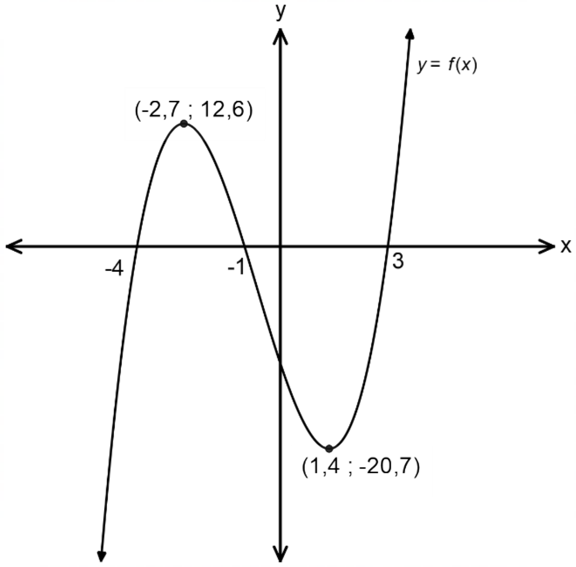
$\dfrac{dl}{dx} = -6x + 8 = 0$

$x = \dfrac{4}{3}$

max length $= -3\left(\dfrac{4}{3}\right)^2 + 8\left(\dfrac{4}{3}\right) + 13$

max length $=$ 18,33 units

8. Consider the diagram below:



a. For what value(s) of t will $f(x) + t = 0$ have exactly one root?

we must drop graph by more than 12,6 or raise it by more than 20,7
so $t < -12,6$ or $t > 20,7$

b. For what value(s) of k if will $f(x + k) = 0$ have only positive roots?

we must shift more than 4 untis to the right
$k < -4$

c. For what value(s) of t will $f(x) + t = 0$ have three distinct roots?

we can drop it down by less than 12,6 or raise it by less than 20,7
$t > -12,6$ and $t < 20,7$
$-12,6 < t < 20,7$

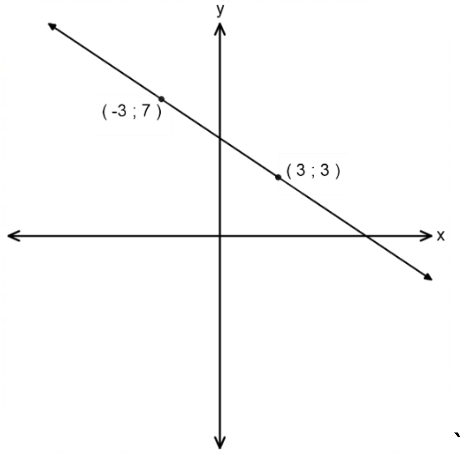
we could also rearrange to:
$f(x) = -t$ and think of the intersection of $y = f(x)$ and $y = -t$
they intersect 3 times when
$-12,6 < t < 20,7$

d. For what value(s) of k if will $f(x + k) = 0$ have 3 roots of the same sign?

we must move it more than 4 to the right or more than 3 to the left
$k < -4$ or $k > 3$

9. Find the equations of the following functions:

a.

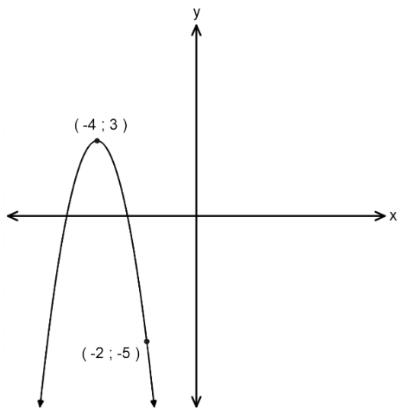


$m = -\frac{4}{6} = -\frac{2}{3}$

$y - 7 = -\frac{2}{3}(x + 3)$

$y = -\frac{2}{3}x + 5$

b.



$y = a(x + 4)^2 + 3$

subst (-2;-5)

$-5 = a(-2 + 4)^2 + 3$

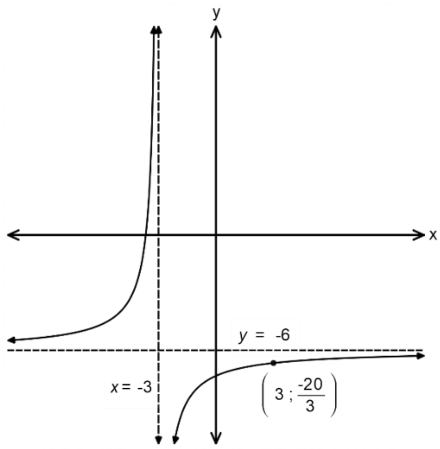
$-8 = 4a$

$a = -2$

$y = -2(x + 4)^2 + 3$

$y = -2x^2 - 16x - 29$

c.



$y = \frac{k}{x + 3} - 6$

subst. $\left(3; -\frac{20}{3}\right)$

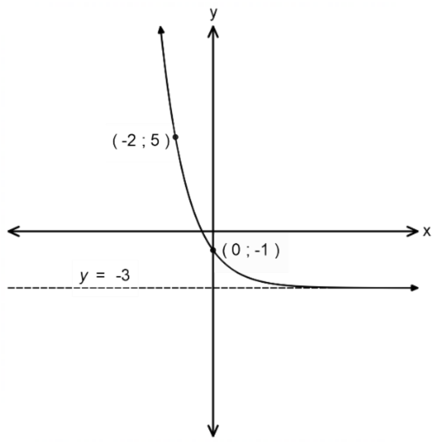
$-\frac{20}{3} = \frac{k}{6} - 6$

$-40 = k - 36$

$k = -4$

$u = -\frac{4}{x + 3} - 6$

d.



$y = ab^x - 3$

subst. (0;-1) so $-1 = ab^0 - 3$ so $a = 2$

$y = 2b^x - 3$

subst. (-2;5) *so* $5 = 2b^{-2} - 3$

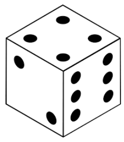
$8 = 2b^{-2}$

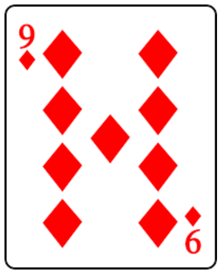
$4 = \frac{1}{b^2}$ so $b = \frac{1}{2}$

$y = 2 \times \left(\frac{1}{2}\right)^x - 3$

Probability and Counting Techniques

1. I toss a die and I draw a card from a well shuffled pack. Determine of:





a. P(even on die and a spade)

$$=\frac{1}{2}\times\frac{1}{4}$$

$$=\frac{1}{8}$$

b. $P(<3$ on die and a black card)

$$=\frac{2}{6}\times\frac{1}{2}$$

$$=\frac{1}{6}$$

c. P(even on die or a red card)

$$=\frac{1}{2}+\frac{1}{2}-\frac{1}{2}\times\frac{1}{2}$$

$$=\frac{3}{4}$$

d. $P(<3$ on die or a diamond)

$$=\frac{2}{6}+\frac{1}{4}-\frac{2}{6}\times\frac{1}{4}$$

$$=\frac{1}{2}$$

2. Events A and B are such that $P(A) = 0{,}4$ and $P(B) = 0{,}5$ and $P(A \cap B) = 0{,}25$

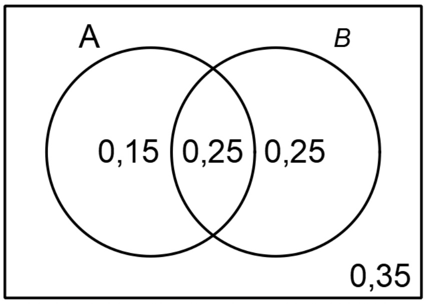
a. Are A and B independent? Justify your answer.

No, since $P(A) \times P(B) = 0{,}2$ and this is $\neq P(A \cap B)$

b. Are A and B mutually exclusive? Justify your answer.

No, since $P(A \cap B) \neq 0$

c. Draw a venn diagram



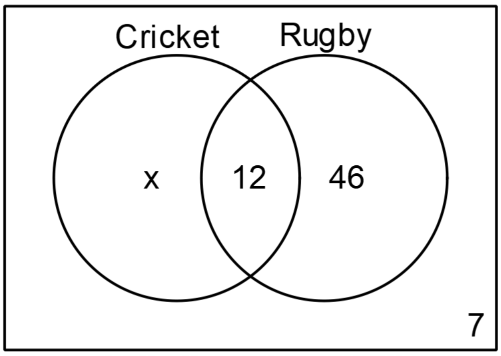
d. Determine $P(A \cup B)$

$P(A \cup B) = 0{,}75$

e. Determine $P\left(A \cap \overline{B}\right)$

$P\left(A \cap \overline{B}\right) = 0{,}15$

3. Consider the venn diagram below:



If $P\left(\text{cricket} \cap \overline{rugby}\right) = \dfrac{3}{16}$ then find x.

$$\frac{x}{x + 12 + 46 + 7} = \frac{3}{16}$$

$$\frac{x}{x + 65} = \frac{3}{16}$$

$$16x = 3x + 195$$

$$13x = 195$$

$$x = 15$$

4. Complete the following table and then use it to determine whether "drinking coca cola" is dependent on gender. You must justify your answer with a calculation.

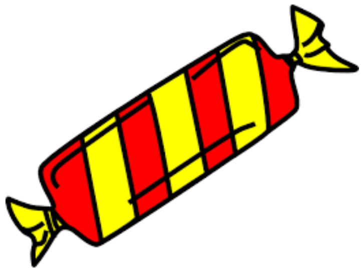
	Drink coke	Don't drink coke	Total
Male	45		
Female			45
Total	72		120

$$P(\text{male} \cap coke) = \frac{45}{120} = 0{,}375$$

$$P(male) \times P(coke) = \frac{75}{120} \times \frac{72}{120} = 0{,}375$$

since these are equal, drinking coca cola is INDEPENDENT of gender

5. A bag contains 11 sweets, some of which are fizzers, and the rest are toffees. I eat two at random. If the probability that at least one is a fizzer is $\frac{52}{55}$ then how many fizzers were there?



suppose f are fizzers then $11-f$ are toffees

P(at least 1 $fizzer$) = $1-P(only$ toffees)

$$1-\frac{11-f}{11}\times\frac{10-f}{10}=\frac{52}{55}$$

$$\frac{110}{110}-\frac{(11-f)(10-f)}{110}=\frac{52}{55}$$

$$\frac{110-(11-f)(10-f)}{110}=\frac{52}{55}$$

$$\frac{110-(110-21f+f^2)}{110}=\frac{52}{55}$$

$$\frac{21f-f^2}{110}=\frac{52}{55}$$

$$21f-f^2=104$$

$$f^2-21f+104=0$$

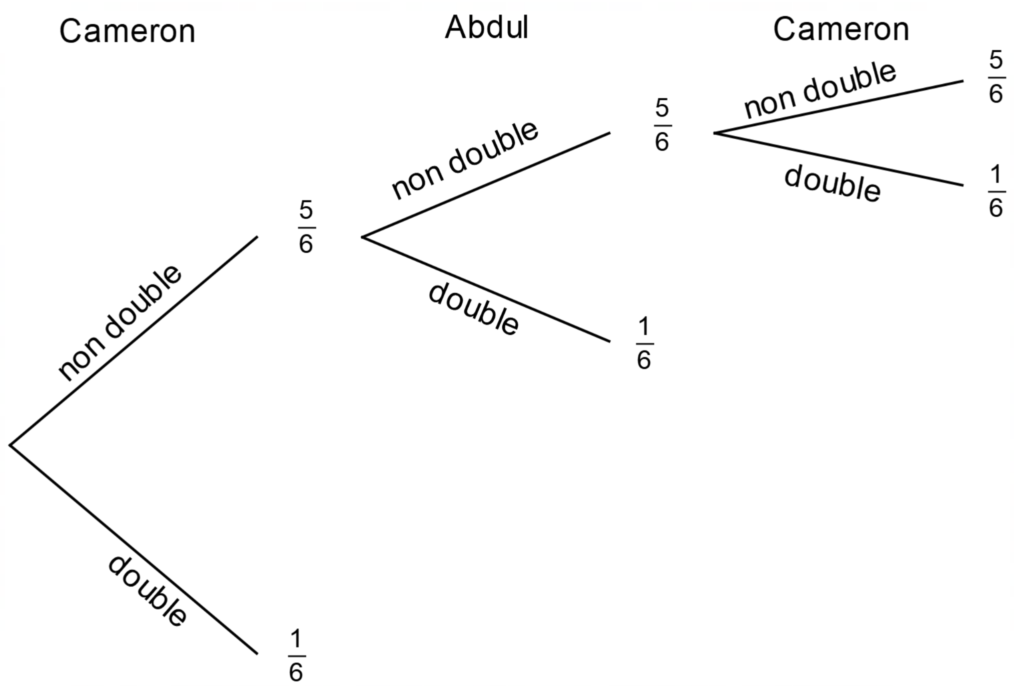
$$(f-13)(f-8)=0$$

$f=13$ (impossible) or 8

so, there were 8 fizzers

6. Cameron and Abdul are playing a game with two dice. They take turns throwing the dice with Cameron starting. Whoever throws a double first is the winner. What is the probability that Cameron wins? Hint: draw a tree diagram.

Express your answer as a percentage to 1 d.p.



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Cameron's chance of winning can be expressed as a geometric series:

$$\frac{1}{6}+\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)\times\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)\times\left(\frac{5}{6}\right)^6\left(\frac{1}{6}\right)+\ .....$$

this is a convergent series since $r=\frac{25}{36}$

$$\text{P(Cameron } winning) = S_\infty = \frac{a}{1-r} = \frac{\frac{1}{6}}{1-\frac{25}{36}}$$

$\text{P(Cameron } winning) = 54{,}5\%$

7. How many different number plates can I make with the following format:

3 alphabetic letters followed by 4 numeric digits.

- To prevent the occurrence of rude words, the five vowels are excluded.
- Letters may be repeated
- The numeric digits may not be repeated.

An example number plate is GRG 9341

How many different number plates are possible?

$total = 21^3 \times 10 \times 9 \times 8 \times 7$

$total = 46\ 675\ 440$

8. Consider the digits 5 , 7 , 2 , 6 and 3. Numbers are formed using some or all of the digits. There is only one of each digit – in other words digits cannot be repeated.

How many 3-digit numbers are possible?

$5 \times 4 \times 3 = 60$

How many numbers are possible altogether?

1 digit $\rightarrow$ 5

2 digits $\rightarrow$ $5 \times 4 = 20$

3 digits $\rightarrow$ 60

4 digits $\rightarrow$ $5 \times 4 \times 3 \times 2 = 120$

5 digits $\rightarrow$ $5 \times 4 \times 3 \times 2 \times 1 = 120$

so, a total of 325

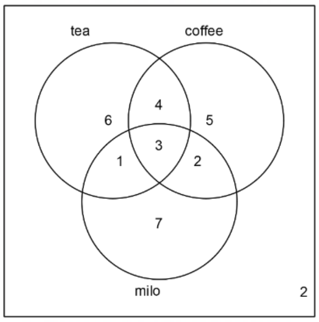
What is the probability that a randomly selected number is bigger than 4000?

It must have 4 or more digits
but if it only has 4 digits then the first digit must be 4 or 5
4 digits starting with 4 $\rightarrow$ $4 \times 3 \times 2 = 24$
4 digits starting with 5 $\rightarrow$ $4 \times 3 \times 2 = 24$
Plus the 5 digit numbers $\rightarrow$ 120

$$P(\text{number} > 4\ 000) = \frac{168}{325}$$

9. Thirty people are asked about their preference of hot drinks (tea, coffee and milo). The results are as follows:

- 3 like all of the drinks
- 7 people like tea and coffee
- 5 people like coffee and milo
- 4 people like tea and milo
- 14 people like tea
- 14 people like coffee
- 13 people like milo



Draw a venn diagram and use it to answer the following questions:

a. How many people drink none of the hot drinks?

b. Calculate the following probabilities:

i. $P\left(\text{tea} \cap \overline{\text{coffee}}\right) = \dfrac{7}{30}$

ii. $P(\text{coffee} \cup milo) = \dfrac{22}{30}$

10. Three boys and five girls go to watch a soccer game. They are seated randomly in 8 adjacent seats. What is the probability that:

a. The 3 boys are seated together?

$$\text{total arrangements} = 8!$$
$$\text{number of ways with boys together} = 6! \times 3!$$
$$P(\text{boys together}) = \frac{6! \times 3!}{8!}$$
$$\underline{P(\text{boys together}) = 10{,}71\%}$$

b. The 5 girls are seated together?

$$\text{total arrangements} = 8!$$
$$\text{number of ways with girls together} = 4! \times 5!$$
$$P(\text{girls together}) = \frac{4! \times 5!}{8!}$$
$$\underline{P(\text{girls together}) = 7{,}14\%}$$

c. The 3 boys are seated together, and the 5 girls are seated together?

$$\text{total arrangements} = 8!$$
$$\text{number of ways with boys together } AND \text{ girls together} = 2! \times 3! \times 5!$$
$$P(\text{Farfield boys together}) = \frac{2! \times 3! \times 5!}{8!}$$
$$\underline{P(\text{Farfield boys together}) = 3{,}57\%}$$

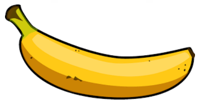
11. How many different 8 letter “words” can be made using the letters NEEDLESS?

$$\frac{8!}{3! \times 2!}$$
$$\underline{= 3\,360}$$

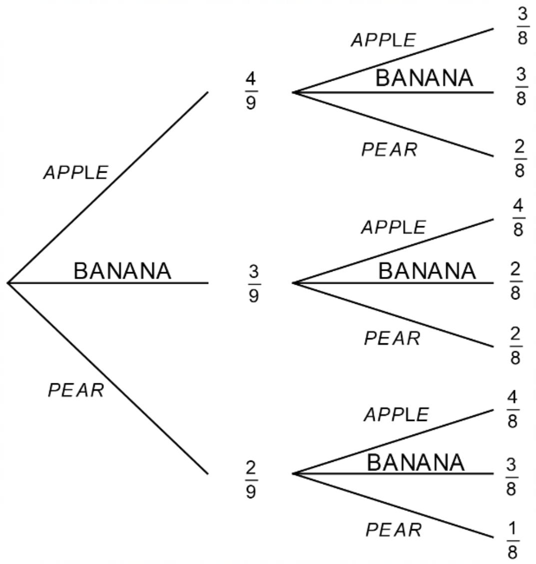
If one is chosen at random then what is the probability that it starts and ends with the same letter?

$$\text{starting and ending with } S \rightarrow \frac{6!}{3!} = 120$$
$$\text{starting and ending with } E \rightarrow \frac{6!}{2!} = 360$$
$$P(\text{start and end with same letter}) = \frac{120 + 360}{3\,360}$$
$$\underline{P(\text{start and end with same letter}) = 14{,}29\%}$$

12. A bag contains 4 apples, 3 bananas and 2 pears. I randomly select two items and eat them.



Draw a tree diagram and use it to answer the questions which follow:



a. $P(\text{at least one } apple) = \frac{4}{9} \times \frac{3}{8} + \frac{3}{9} \times \frac{4}{8} + \frac{2}{9} \times \frac{4}{8}$

$P(\text{at least one } apple) = 44{,}4\%$

b. $P(\text{an apple and a banana in any } order) = \frac{4}{9} \times \frac{3}{8} + \frac{3}{9} \times \frac{4}{8}$

$P(\text{an apple and a banana in any } order) = 33{,}3\%$

13. I have 10 shirts. Three have short sleeves and are identical. Two others have short sleeves but are different from one another and different to the other two. The remaining five shirts have long sleeves.

a. In how many different ways can the shirts be arranged?

$\frac{10!}{3!} = 604\ 800$

b. What is the probability that the short-sleeved shirts end up together?

short sleeved shirts can be arranged together in $\frac{5!}{3!} = 20$ ways

$P(\text{short sleeved } together) = \frac{20}{604\ 800}$