Grade 12 Mathematics

Paper 2 Revision Workshop

29th August 2026

NAME: WORKED SOLUTIONS

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Data Handling

1. Consider the following data set and answer the questions which follow:

9	1 8
8	2 2 4 5 6 6 7
7	0 2 3 5 8 9
6	3 5 6
5	2 4 4 4
4	5

a. What is the mode?

54

b. What is the range?

range = $98 - 45 = 53$

c. What is the mean to 2 d.p.?

$\overline{x} = 73{,}09$

a. Use your calculator to find the standard deviation to 3 d.p.

$\sigma = 13{,}997$

b. Give the 5 number summary.

$\frac{n+1}{4} = 6$ *so*

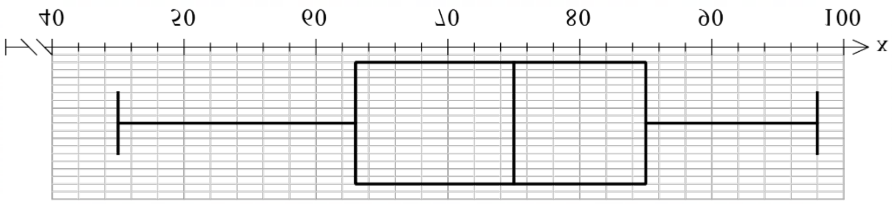
Q_1 is in position 6

Q_2 is in position 12

Q_3 is in position 18

5 number summary is 45 ; 63 ; 75 ; 85 ; 98

c. Plot a box and whisker on the axes provided.



g. Comment on the symmetry / skewness of the data

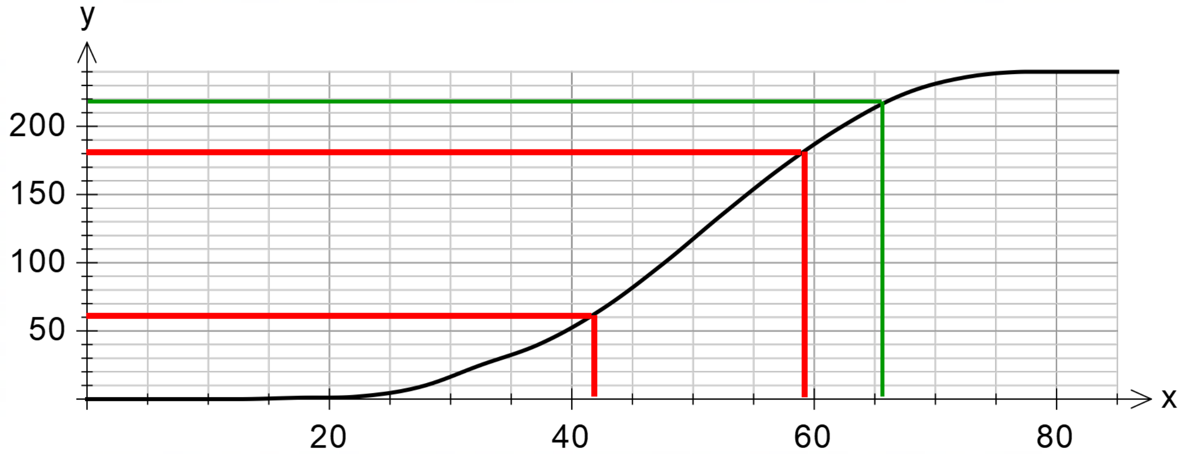
skewed slightly left *OR* slightly negatively skewed

2. Determine the standard deviation (2 d.p.) of the following four numbers by completing the table:

2 ; 6 ; 7 ; 13 $\quad\quad\quad \overline{x} = 7$

x_i	$x_i - \overline{x}$	$(x_i - \overline{x})^2$
2	-5	25
6	-1	1
7	0	0
13	6	36
	$\sum_{i=1}^{4}(x_i - \overline{x})^2$	62
	Variance	$\frac{62}{4} = 15,5$
	σ	$\sqrt{15,5} = 3,94$

3. Consider the following cumulative frequency diagram representing exam marks and answer the questions which follow:



a. How many candidates were there?

240

b. Estimate the inter-quartile range showing where you have taken your readings.

See orange lines

Q_1 at $60 \approx 41$

Q_3 at $180 \approx 59$

Inter-quartile range ≈ 18

c. It is decided that the top 10% of candidates will receive an award for outstanding achievement. Estimate the mark required to attain this award showing where you have taken your reading.

See green lines 90% of $240 = 216 \rightarrow \approx 66\%$

4. A data set has $\sum_{i=1}^{n}\left(x_i-\overline{x}\right)^2 = 200$. If the standard deviation is 2 then how many data items are there?

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i-\overline{x}\right)^2}{n} = \frac{200}{n}$$

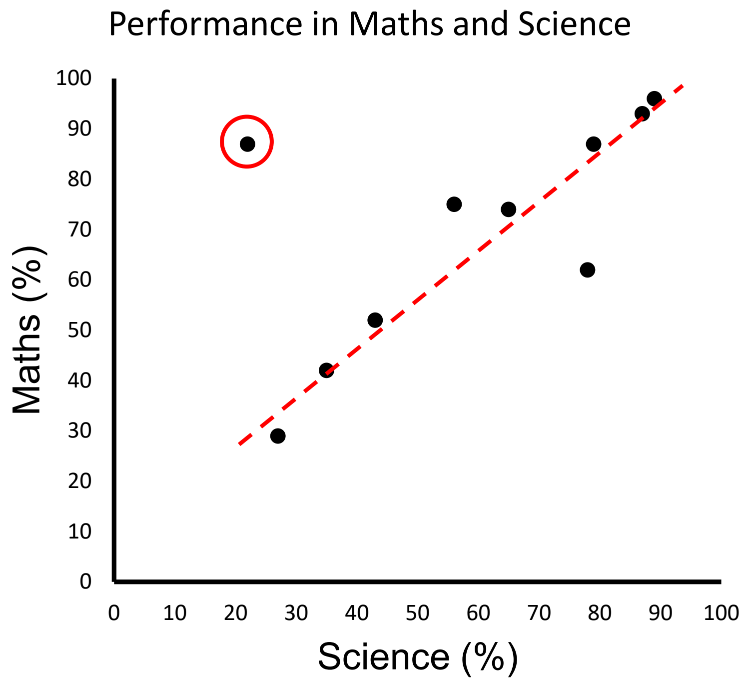
$$\sigma = \sqrt{\frac{200}{n}} = 2$$

$$\therefore \frac{200}{n} = 4$$

$$\therefore 200 = 4n$$

$$\therefore n = 50$$

5. Consider this scatter plot showing the Science and Maths marks of 10 learners



a. Comment on the nature of the relationship and give an estimate for the correlation coefficient.

It seems that people who do better at Science do better at Maths!

There is a fairly strong, positive correlation

$r \approx 0{,}65$

b. Draw in a line of best fit by eye.

c. One point could be considered an outlier. Circle it.

d. What would be the effect of removing the outlier on:

i. the slope of the least squares regression line?

It would be slightly less steep – in other words it would decrease

ii. the correlation coefficient?

It would go closer to 1 – in other words it would increase

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6. Consider a set of exam marks which have been recorded as grouped data.

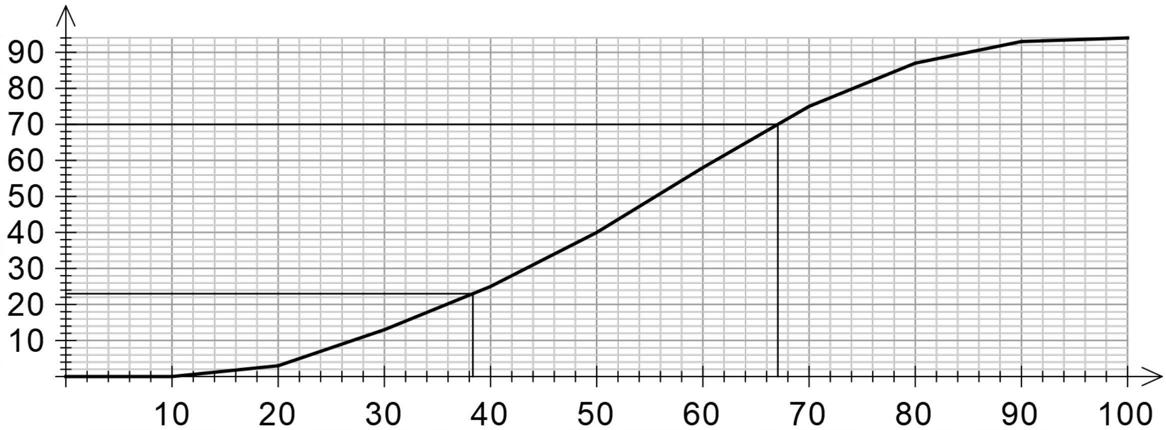
Mark	Frequency	Cumulative Frequency
0-10	0	0
10-20	3	3
20-30	10	13
30-40	12	25
40-50	15	40
50-60	18	58
60-70	17	75
70-80	12	87
80-90	6	93
90-100	1	94

a. Determine an **estimate** for the mean, showing an indication of how you have worked it out. Explain why it is an **estimate**.

$$\bar{x} = \frac{(15 \times 3) + (25 \times 10) + \ldots + (95 \times 1)}{94} = \frac{4\,990}{94} = 53{,}1\%$$

it is an *ESTIMATE* since we don't have the ACTUAL data. We are using the mid-pt as the 'best guess' of the average performance of the individuals in each group

b. Draw a cumulative frequency curve on the axes provided. Label your axes. Use your curve to estimate the inter-quartile range showing where you have taken your readings.



inter-quartile range = $67 - 38 = 29$

7. As a coffee shop owner you have been recording sales vs temperature.

Temperature (^{o}C)	No. of cups of coffee sold
30	4
28	4
25	11
22	25
20	28
18	30
14	48
12	49
10	51

a. Determine the equation of the least squares regression line giving both parameters to 2 d.p. Let x represent temperature and y the number of cups of coffee sold.

$y = -2{,}634x + 80{,}156$

b. Determine the correlation coefficient and describe in words what it tells us about the relationship between temperature and coffee sales.

The correlation coefficient is -0,987
This is a *VERY* strong negative correlation
As temperature increases coffee sales decrease

a. Use your least squares regression line to predict coffee sales when the temperature is 23,5^{o}C and comment on the reliability of this.

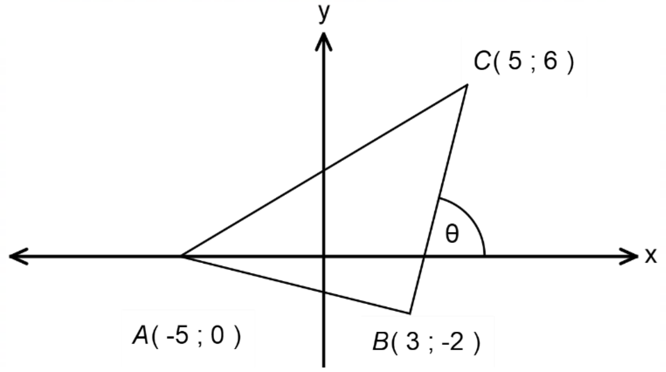
we get 18,257 so 18 cups
Given the strength of the relationship and that we are *INTERPOΛTING*
this is likely highly reliable

b. The weather forecast for tomorrow is 5^{o}C. Can you rely on your least squares regression line to predict sales? Why?

this is extrapolation – highly unreliable.
We don't know that the trend is maintained.
If it gets too cold people won't leave their homes

Analytical Geometry

1. Consider the diagram below:



a. Show that $\triangle ABC$ is right-angled and isosceles.

$mAB = \dfrac{0+2}{-5-3} = -\dfrac{1}{4}$ and $mBC = \dfrac{6+2}{5-3} = 4$

since $mAB \times mBC = -1$ we have $AB \perp BC$

now $AB = \sqrt{(-5-3)^2 + (-2-0)^2} = \sqrt{68}$

and $BC = \sqrt{(5-3)^2 + (6-(-2))^2} = \sqrt{68}$

b. Hence, or otherwise, find area $\triangle ABC$

Area $\triangle ABC = \dfrac{1}{2} \times base \times ht$

Area $\triangle ABC = \dfrac{1}{2} \times \sqrt{68} \times \sqrt{68}$

Area $\triangle ABC = 34u^2$

c. Find D if ABCD (in that order) is a rhombus.

$D(-3;8)$

a. Find θ to one decimal place.

$mBC = 4$ so $\tan\theta = 4$ and $\theta = \arctan(4) = 76°$

b. Find a circle which has centre C and **touches** the *y*-axis

$(x-5)^2 + (y-6)^2 = 25$

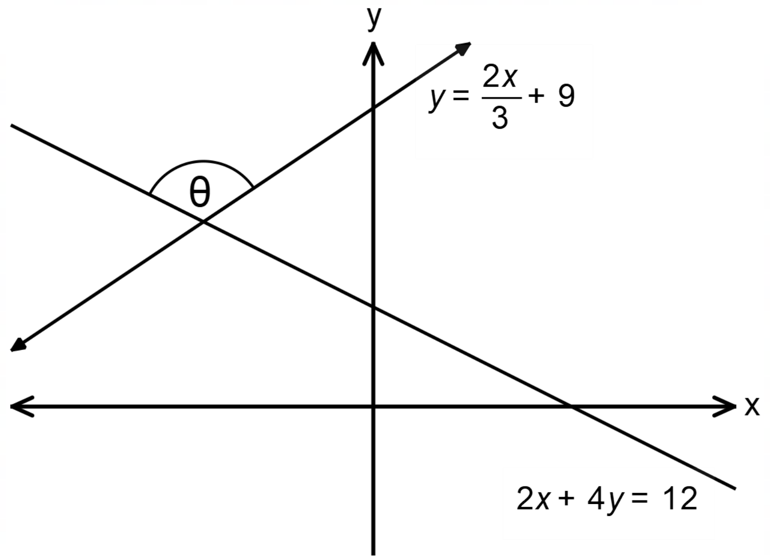
c. Find a point E such that B is the mid-point of AE.

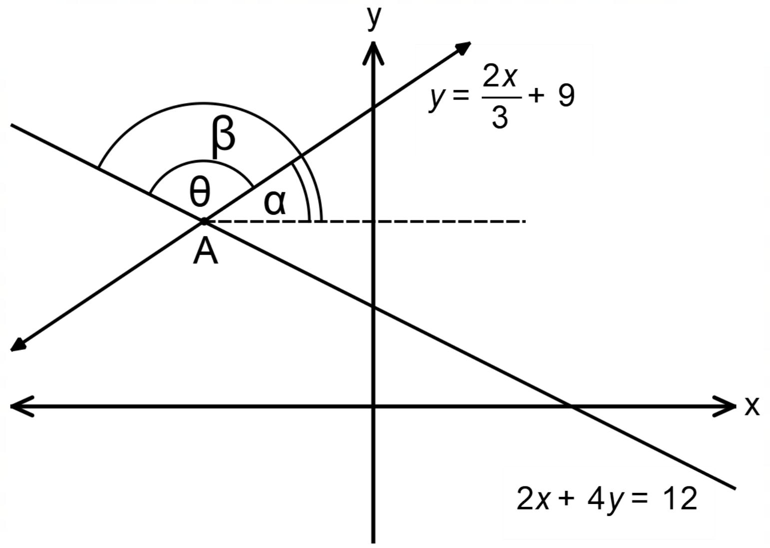
$E(11;-4)$

2. A circle has equation $x^2 - 4x + y^2 + 6y = 12$. Find a second circle with centre (-10;6) which **touches** the first circle.

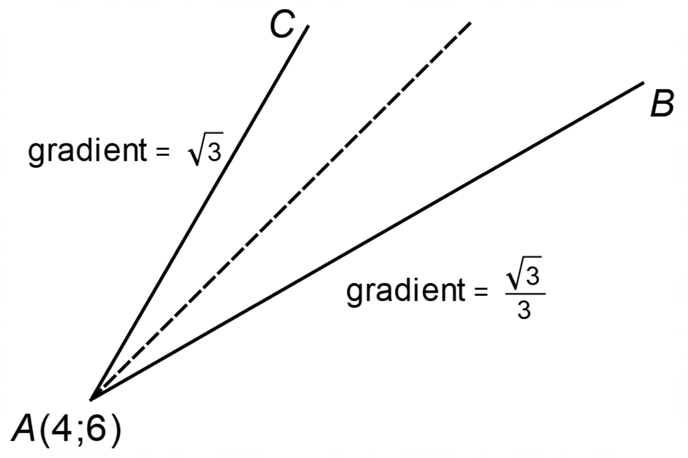
$x^2 - 4x + y^2 + 6y = 12$
$(x-2)^2 + (y+3)^2 = 25$
so, centres are (2;-3) and (-10;6)
the distance between the centres is $\sqrt{12^2 + 9^2} = 15$
first circle has a radius of 5 so second must have a radius of 10
$\therefore (x+10)^2 + (y-6)^2 = 100$

3. Find the size of θ in the diagram below:





4. Find the equation of the dashed line in the diagram below if it bisects angle $C\hat{A}B$



$C\hat{A}B = \arctan\sqrt{3} - \arctan\left(\frac{\sqrt{3}}{3}\right)$

$C\hat{A}B = 30$

so, dashed line is at an angle of $45°$

with a gradient of $\tan 45° = 1$

$y - 6 = (x - 4)$

$\underline{y = x + 2}$

5. A tangent is drawn to the circle $x^2 + y^2 = 6x - 4y$ from the point $A(8;12)$. What is the distance from A to the point of contact of the tangent with the circle?

$x^2 + y^2 = 6x - 4y$

$x^2 + y^2 - 6x + 4y = 0$

$(x-3)^2 + (y+2)^2 = 13$

Centre $O(3;-2)$

suppose pt of contact is B

then $OB \perp BA$ (radius $\perp$ tangent)

$\therefore AB^2 - OB^2 = OA^2$ (Pythagoras)

$\therefore (5^2 + 14^2) - 13 = OA^2$

$\therefore 208 = OA^2$

$\therefore OA = \sqrt{208} \approx 14{,}42$ units

6. Find a point which is $\frac{2}{5}$ of the way between $A(-2;4)$ and $B(8;-16)$

The gap in the xs is 10 and in the ys is 20

$\frac{2}{5}$ of 10 is 4 so, we must add this to the x-coordinate of A

and $\frac{2}{5}$ of 20 is 8 so, we must subtract this from the y-coordinate of A

The required point is (2;-4)

7. Consider the circles $x^2 + y^2 + 4x - 8y = 60$ and $x^2 + y^2 - 16x + 32y = -140$

a. Prove that they **touch** one another.

$x^2 + y^2 + 4x - 8y = 60$ and $x^2 + y^2 - 16x + 32y = -140$

$(x+2)^2 + (y-4)^2 = 60 + 4 + 16$ and $(x-8)^2 + (y+16)^2 = -140 + 64 + 256$

$(x+2)^2 + (y-4)^2 = 80$ and $(x-8)^2 + (y+16)^2 = 180$

so, the centres are (-2;4) and (8;-16)

and the radii are $\sqrt{80} = 4\sqrt{5}$ and $\sqrt{180} = 6\sqrt{5}$

distance between centres is $\sqrt{10^2 + 20^2} = \sqrt{500} = 10\sqrt{5}$

now sum of *radii* $= 10\sqrt{5}$

since these are the same the circles *TOUCH*

b. Now, find the point at which they intersect. Hint: look at Q6!

the centres are the same points as in Question 8 and

the radii are in the ratio – 4 : 6 = 2 : 3

so the point of intersection is $\frac{2}{5}$ of the way beetween the centres at (2;-4)

8. Find the equation of the tangent to the circle $(x+3)^2 + (y-2)^2 = 25$ at the point (1;5)

centre is (-3;2)

the gradient of the *radius* $= \frac{3}{4}$

so gradient of tangent $= -\frac{4}{3}$ (rad $\perp$ tan)

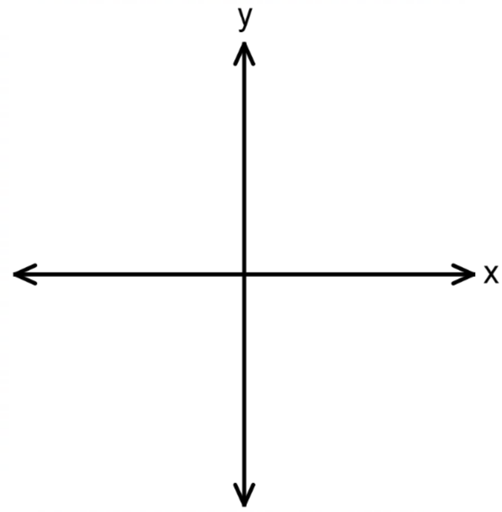
$y - 5 = -\frac{4}{3}(x-1)$

$y = -\frac{4}{3}x + \frac{19}{3}$

Trigonometry

1. Use the given information to draw a sketch. Label θ and find the other two trigonometric ratios and θ.

a. $\cos\theta = -\frac{2}{3}$ and $\sin\theta > 0$



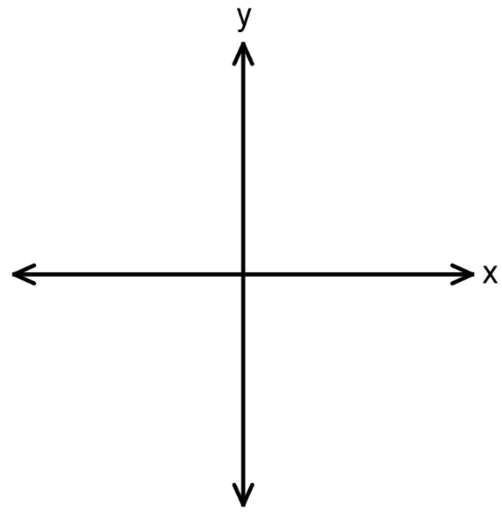
$\sin\theta = \frac{\sqrt{5}}{3}$

$\tan\theta = \frac{\sqrt{5}}{-2}$

key $\angle = \sin^{-1}\left(\frac{\sqrt{5}}{3}\right) = 48{,}2°$

$\therefore \theta = 180° - 48{,}2° = 131{,}8°$

b. $\tan\theta = -2$ and $\cos\theta > 0$



$\sin\theta = -\frac{2}{\sqrt{5}}$

$\cos\theta = \frac{1}{\sqrt{5}}$

key $\angle = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = 63{,}4°$

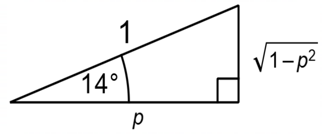
$\therefore \theta = 360° - 63{,}4° = 296{,}6°$

2. If $\cos 14° = p$ then determine the following in terms of p:

a. $\cos 166°$
$= \cos(180° - 14°)$
$= -\cos 14°$
$= -p$

b. $\sin 104°$
$= \sin(90° + 14°)$
$= \cos 14°$
$= p$

c. $\tan 194°$
$= \tan(180° + 14°)$
$= -\tan 14°$



$= -\frac{\sqrt{1-p^2}}{p}$

d. $\cos(-28°)$
$= \cos 28°$
$= \cos 2(14°)$
$= 2\cos^2 14° - 1$
$= 2p^2 - 1$

e. $\cos 187°$

$= \cos(180° + 7°)$

$= -\cos 7°$

now $p = \cos 14° = 2\cos^2 7° - 1$

$\therefore 2\cos^2 7° - 1 = p$

$\therefore 2\cos^2 7° = p + 1$

$\therefore 2\cos^2 7° = \dfrac{p+1}{2}$

$\therefore \cos^2 7° = \dfrac{p+1}{2}$

$\therefore \cos 7° = \sqrt{\dfrac{p+1}{2}}$

$\therefore \cos 187° = -\sqrt{\dfrac{p+1}{2}}$

2. Simplify the following without a calculator:

a. $\dfrac{\sin(180° - \theta)\tan(180 - \theta)}{\tan(-\theta)\sin(90° + \theta)}$

$= \dfrac{\sin\theta\tan\theta}{-\tan\theta\cos\theta}$

$= \dfrac{\sin\theta\cancel{\tan\theta}^{1}}{-\cancel{\tan\theta}^{1}\cos\theta}$

$= -\tan\theta$

b. $\dfrac{\sin 240°\cos 180°\sin 17°}{\tan 225°\cos(-150°)\cos 107°}$

$= \dfrac{\sin(180° + 60°)(-1)\sin 17°}{\tan(180° + 45°)(-\cos 30°)\cos(90 + 17°)}$

$= \dfrac{-\sin 60°(-1)\sin 17°}{(\tan 45°)(-\cos 30°)(-\sin 17°)}$

$= \dfrac{-\sin 60°(-1)\sin 17°}{(\tan 45°)(-\cos 30°)(-\sin 17°)}$

$= \dfrac{\frac{\sqrt{3}}{2}\cancel{\sin 17°}^{1}}{1\left(-\frac{\sqrt{3}}{2}\right)\left(-\cancel{\sin 17°}^{1}\right)}$

$= 1$

c. $\dfrac{\cos 20°\cos 37° - \sin 20°\sin 37°}{\sin 33°}$

$= \dfrac{\cos(20° + 37°)}{\sin 33°}$

$= \dfrac{\cos 57°}{\sin(90° - 57°)}$

$= \dfrac{\cancel{\cos 57°}^{1}}{\cancel{\sin 57°}^{1}}$

$= 1$

d. $\dfrac{\sin 140°}{\sin 160°\cos 200°}$

$= \dfrac{\sin(180° - 40°)}{\sin(180° - 20°)\cos(180° + 20°)}$

$= \dfrac{\sin 40°}{\sin 20° \times -\cos 20°}$

$= \dfrac{\sin 2(20°)}{\sin 20° \times -\cos 20°}$

$= \dfrac{2\cancel{\sin 20°}^{1}\cancel{\cos 20°}^{1}}{\cancel{\sin 20°}^{1} \times -\cancel{\cos 20°}^{1}}$

$= -2$

e. $\dfrac{\sin 23°\cos 22° + \cos 23°\sin 22°}{\cos 49°\cos 4° + \sin 49°\sin 4°}$

$= \dfrac{\sin(23° + 22°)}{\cos(49° - 4°)}$

$= \dfrac{\sin 45°}{\cos 45°}$

$= \tan 45°$

$= 1$

3. Prove that: $\dfrac{\cos\theta}{1 + \sin\theta} + \tan\theta = \dfrac{1}{\cos\theta}$

$LHS = \dfrac{\cos\theta}{1 + \sin\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta}{(\cos\theta)(1 + \sin\theta)} + \dfrac{(\sin\theta)(1 + \sin\theta)}{(\cos\theta)(1 + \sin\theta)}$

$= \dfrac{\cos^2\theta + \sin\theta + \sin^2\theta}{(\cos\theta)(1 + \sin\theta)}$

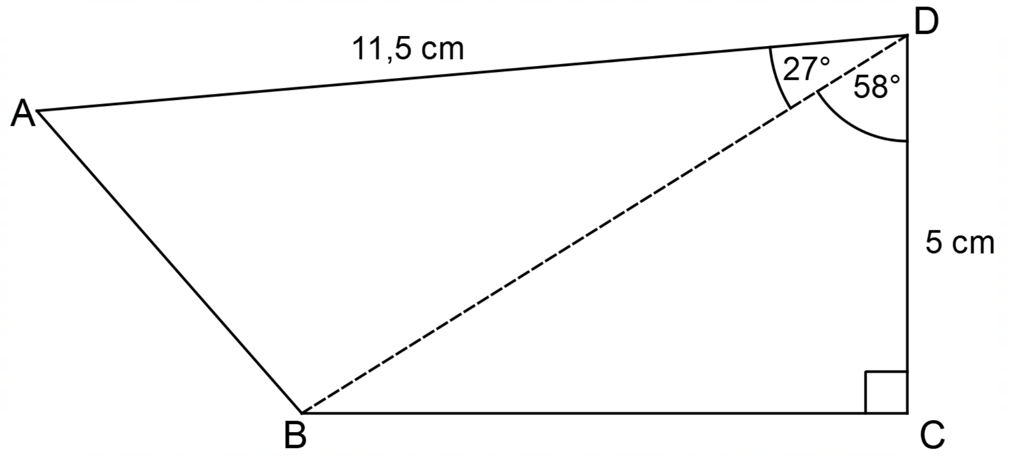
$= \dfrac{1 + \sin\theta}{(\cos\theta)(1 + \sin\theta)}$

$= \dfrac{\cancel{1 + \sin\theta}^{1}}{(\cos\theta)\cancel{(1 + \sin\theta)}^{1}}$

$= \dfrac{1}{\cos\theta}$

$= RHS$

4. Find the perimeter of the shape below. Remember: don't include the dashed line!



$\cos 58° = \dfrac{5}{BD}$ and $\tan 58° = \dfrac{BC}{5}$

$BD = \dfrac{5}{\cos 58°} = 9{,}44$ and $BC = 5\tan 58° = 8$

now $AB^2 = BD^2 + AD^2 - 2(BD)(AD)\cos 27°$

$\therefore AB^2 = 9{,}44^2 + 11{,}5^2 - 2(9{,}44)(11{,}5)\cos 27°$

$\therefore AB = \sqrt{9{,}44^2 + 11{,}5^2 - 2(9{,}44)(11{,}5)\cos 27°}$

$\therefore AB = 5{,}28$

Perimeter = $5 + 8 + 5{,}28 + 11{,}5$

Perimeter = 29,8 cm

5. Solve the following equations for $\theta \in [0°; 360°]$:

a. $2\cos\theta + 3 = 2$

$2\cos\theta = -1$

$\cos\theta = -\dfrac{1}{2}$

key $\angle = \cos^{-1}\left(\dfrac{1}{2}\right) = 60°$

quadrants 2 and 3

$\theta = 120°$ or $240°$

b. $\sin\theta = 2\cos\theta$

$\tan\theta = 2$

key $\angle = \tan^{-1}(2) = 63{,}4°$

quadrants 1 and 3

$\therefore \theta = 63{,}4°$ or $243{,}4°$

c. $\tan(2\theta + 15°) = -1$

key $\angle = \tan^{-1}(1) = 45°$

quadrants 2 and 4

$\therefore \theta = 135°$ or $315°$

d. $2\sin\theta\cos\theta = \dfrac{1}{2}$

$\sin 2\theta = \dfrac{1}{2}$

key $\angle = \sin^{-1}\left(\dfrac{1}{2}\right) = 30°$

quadrants 1 and 2

$2\theta = 30° + 360k$ or $2\theta = 150° + 360k$

$\theta = 15° + 180k$ or $\theta = 75° + 180k$

$\therefore \theta = 15°$ or $195°$ or $75°$ or $255°$

e. $\sin\theta = \cos 72°$

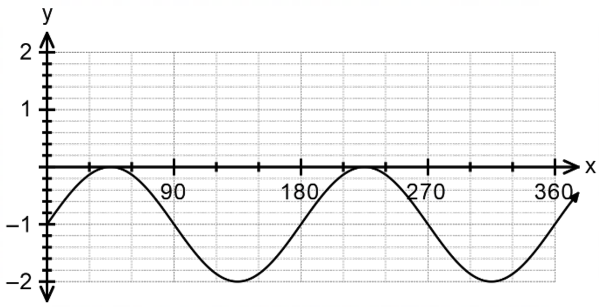
$\sin\theta = \cos(90° - 18°)$

$\sin\theta = \sin 18°$

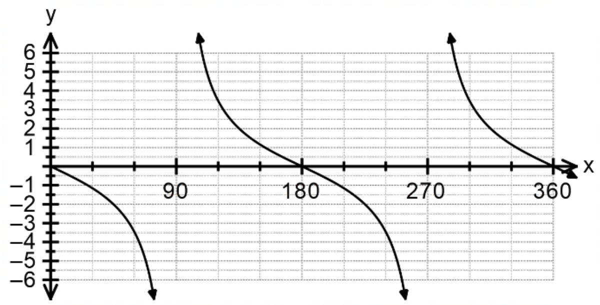
$\therefore \theta = 18°$ or $162°$

6. Sketch the following graphs on the axes provided:

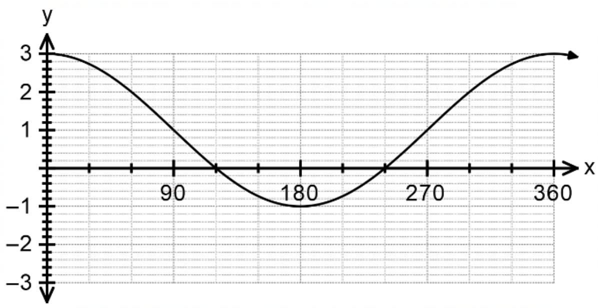
a. $y = \sin 2\theta - 1$



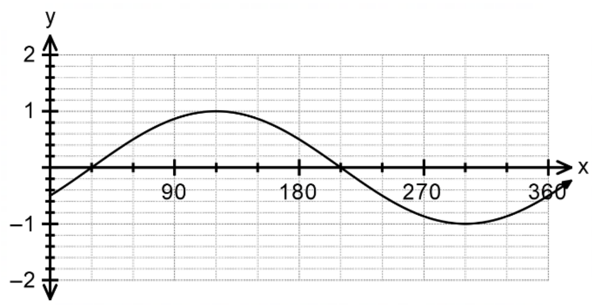
b. $y = -2\tan\theta$



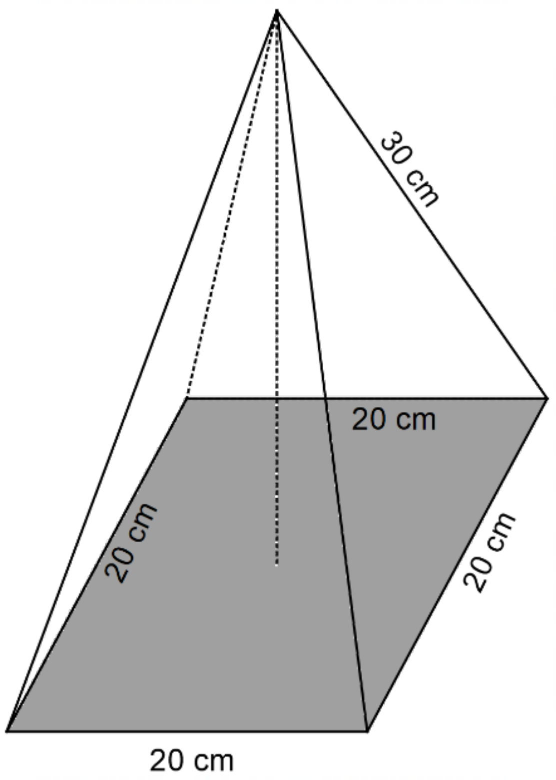
c. $y = 2\cos\theta + 1$

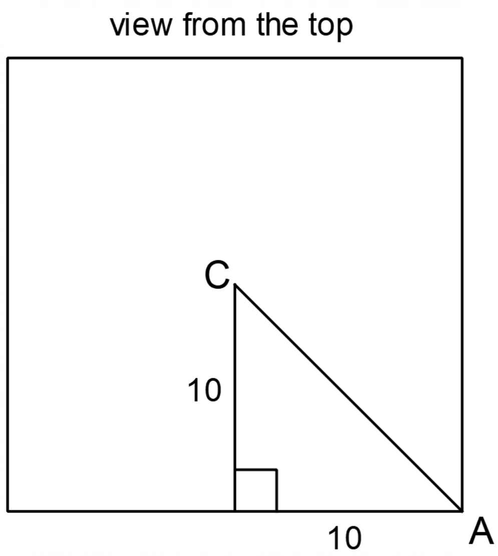


d. $y = \sin\theta\cos 30° - \cos\theta\sin 30°$



7. Consider the pyramid below. It has a square base with sides of 20 cm. If the slant height (s) is 12 cm, then how high is the pyramid?





$AC^2 = 10^2 + 10^2 = 200$

now $AB^2 = 900$

$\therefore BC^2 = 900 - 200 = 500$

$\therefore BC = \sqrt{500}$

$\therefore BC \approx 22{,}4$ cm

8. Prove the following identities:

a. $$\frac{(\sin\theta + \cos\theta)^2 - 1}{\cos^2\theta} = 2\tan\theta$$

$$\frac{(\sin\theta + \cos\theta)^2 - 1}{\cos^2\theta} = 2\tan\theta$$

$$LHS = \frac{(\sin\theta + \cos\theta)^2 - 1}{\cos^2\theta}$$

$$= \frac{\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 1}{\cos^2\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\cos^2\theta}$$

$$= \frac{2\sin\theta}{\cos\theta}$$

$$= 2\tan\theta$$

RHS

It will be *undefined when* $\cos\theta = 0$ *or when* tan *is undefined which is* the *same so, when* $\theta = 90° + 180k$, *for* $k \in \mathbb{Z}$

b. $$\frac{\cos2\theta - \cos\theta}{\sin2\theta + \sin\theta} = \frac{\cos\theta - 1}{\sin\theta}$$

$$LHS = \frac{2\cos^2\theta - 1 - \cos\theta}{2\sin\theta\cos\theta + \sin\theta}$$

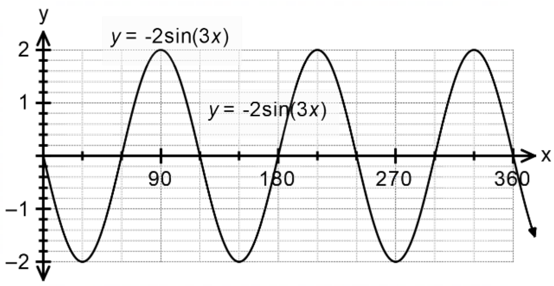
$$= \frac{(2\cos\theta + 1)(\cos\theta - 1)}{\sin\theta(2\cos\theta + 1)}$$

$$= \frac{(\cos\theta - 1)}{\sin\theta}$$

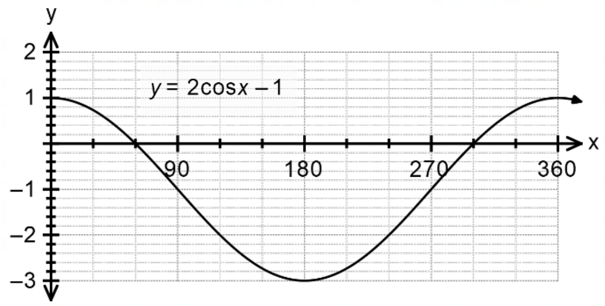
$= RHS$

9. Find the equations of the following functions:

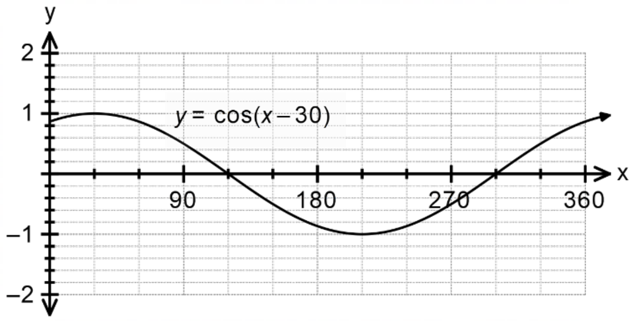
a.



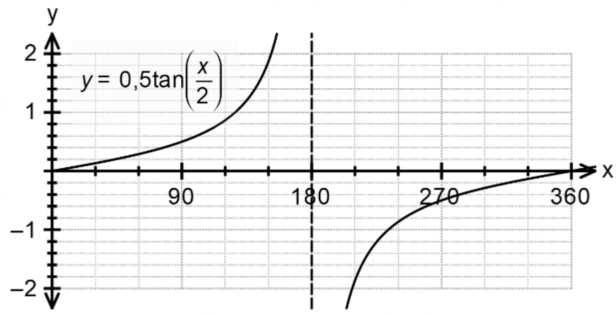
b.



c.



d.



10. Prove the following identities:

a. $\dfrac{\sin(2x+y)+\sin(2x-y)}{\cos x\cos y} = 4\sin x$

$$LHS = \frac{\sin 2x\cos y + \cos 2x\sin y + \sin 2x\cos y - \cos 2x\sin y}{\cos x\cos y}$$

$$= \frac{2\sin 2x\cos y}{\cos x\cos y}$$

$$= \frac{2\times 2\sin x\cos x\cos y}{\cos x\cos y}$$

$$= \frac{4\sin x\cancel{\cos x}^{1}\cancel{\cos y}^{1}}{\cancel{\cos x}^{1}\cancel{\cos y}^{1}}$$

$$= 4\sin x$$

$$= RHS$$

b. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$$LHS = \sin(2\theta+\theta)$$

$$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$$

$$= 2\sin\theta\cos\theta\cos\theta + (1-2\sin^2\theta)\sin\theta$$

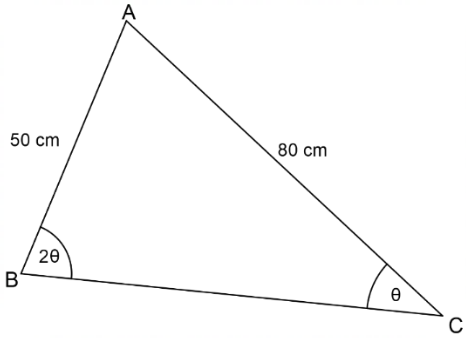
$$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$$

$$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$$

$$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$$

$$= 3\sin\theta - 4\sin^3\theta$$

11. Find the area of ΔABC



$$\frac{80}{\sin(2\theta)} = \frac{50}{\sin\theta}$$

$$\frac{80}{2\sin\theta\cos\theta} = \frac{50}{\sin\theta}$$

$$\therefore 80\sin\theta = 100\sin\theta\cos\theta$$

$$\therefore 80\sin\theta - 100\sin\theta\cos\theta$$

$$\therefore 20\sin\theta(4 - 5\cos\theta) = 0$$

$$\therefore \sin\theta = 0° \text{ (impossible in a triangle) or } \cos\theta = \frac{4}{5}$$

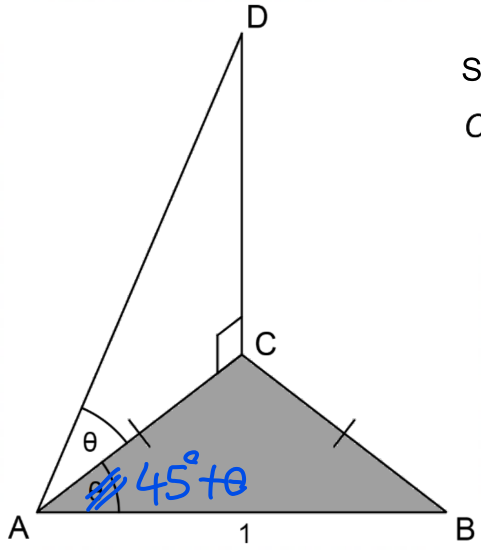
$$\therefore \theta = \cos^{-1}\left(\frac{4}{5}\right) = 36{,}87$$

$$\therefore \hat{A} = 180' - 3\theta = 69{,}39$$

$$\therefore \Delta ABC = \frac{1}{2}(50)(80)\sin(69{,}39°)$$

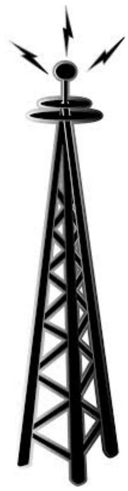
$$\therefore \Delta ABC = 1\,872\text{cm}^2$$

12. Consider the diagram in which ΔABC is an isosceles triangle in the horizontal plane. DC is a vertical tower. The angle of elevation of D from A is θ. $C\hat{A}B = \cancel{\theta}$ and $AB = 1$ unit.
45° + θ



Show that the height of the tower

$$CD = \frac{1-\cos^2\theta}{2\cos^3\theta - \cos\theta}$$



Apologies $C\hat{A}B = 45° + \theta$

$$\frac{1}{\sin(90° - 2\theta)} = \frac{AC}{\sin\theta}$$

$$AC\cos 2\theta = \sin\theta$$

$$AC = \frac{\sin\theta}{\cos 2\theta}$$

$$\tan\theta = \frac{CD}{AC}$$

$$CD = \tan\theta AC$$

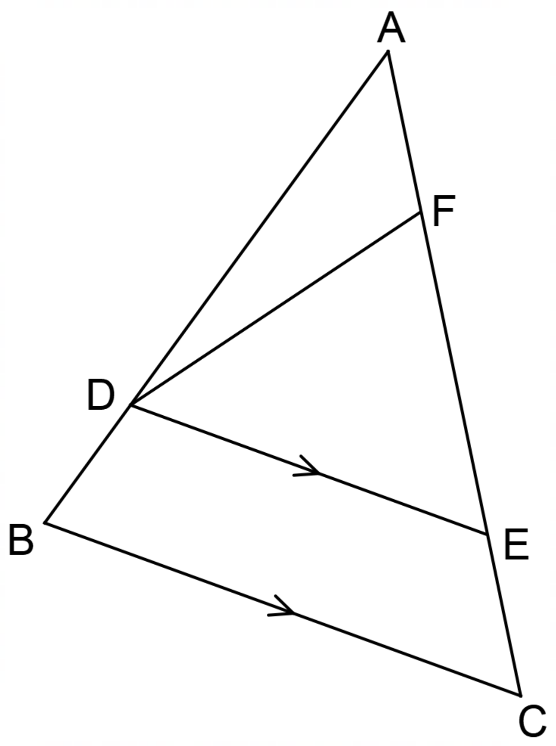
$$CD = \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\cos 2\theta}$$

$$CD = \frac{\sin^2\theta}{\cos\theta \times (2\cos^2\theta - 1)}$$

$$CD = \frac{1-\cos^2\theta}{2\cos^3\theta - \cos\theta}$$

Euclidean Geometry

1. Consider the diagram below. $\frac{AD}{AB} = \frac{3}{4}$ and $\frac{AF}{FE} = \frac{1}{2}$.



a. Determine $\frac{FE}{FC}$ giving reasons.

$\frac{AE}{AC} = \frac{AD}{AB} = \frac{3}{4}$ (proportional intercept thm)

so, let $AE = 3p$ and $EC = p$

no $\frac{AF}{FE} = \frac{1}{2}$ so $AF = p$ and $FE = 2p$

$\therefore \frac{FE}{FC} = \frac{2p}{3p} = \frac{2}{3}$

b. Use the fact that $\Delta AED ||| \Delta ACB$ to determine $\frac{\text{area } \Delta AED}{\text{area } \Delta ACB}$ justifying your answer.

$$\frac{\text{area } \Delta AED}{\text{area } \Delta ACB} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

since areas of similar shapes are in a ratio equal to the square of the ratio of their sides.

c. Determine $\frac{\text{area } \Delta AFD}{\text{area } \Delta AED}$ giving reason(s)

$$\frac{\text{area } \Delta AFD}{\text{area } \Delta AED} = \frac{1}{3} \text{ (shared height)}$$

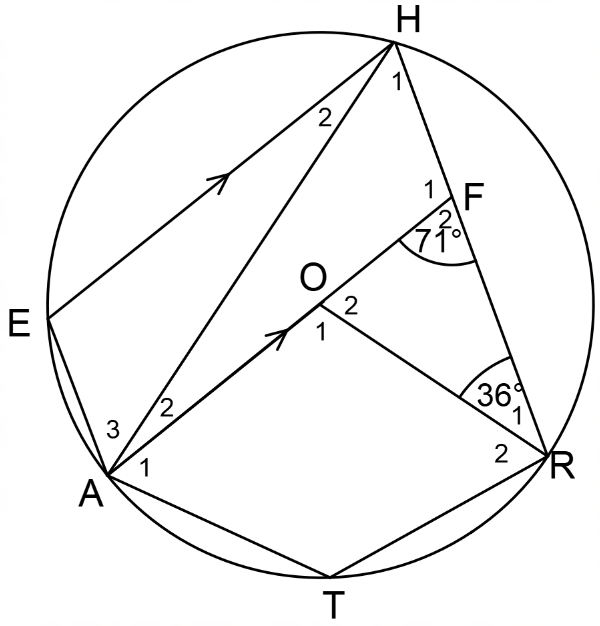
d. Hence, or otherwise, determine $\frac{\text{area } \Delta AFD}{\text{area } \Delta ACB}$

let area $\Delta AFD = a$
then area $\Delta AED = 3a$
now, from b. let area $\Delta AED = 9m$
then area $\Delta ABC = 16m$
now we have $3a = 9m$
so $a = 3m$

$$\therefore \frac{\text{area } \Delta AFD}{\text{area } \Delta ACB} = \frac{3m}{16m} = \frac{3}{16}$$

2. In the diagram O is the centre of circle HEATR. $AOF \parallel EH$. $\hat{F}_2 = 71°$ and $\hat{R}_1 = 36°$.

Calculate, with reasons:



a. $\hat{O}_1 = 107°$ (ext. $\angle$ of Δ)

b. $\hat{H}_1 = 53{,}5°$ ($\angle$ at centre)

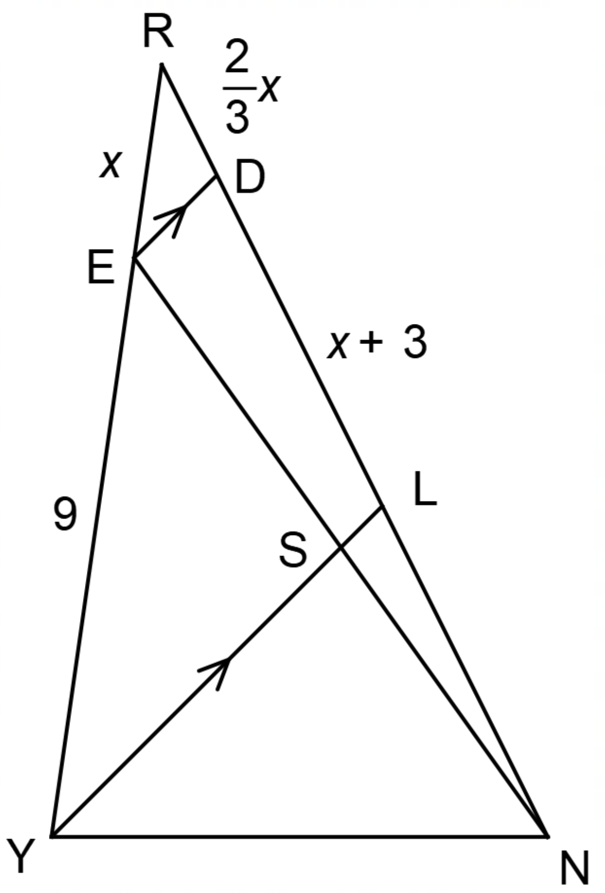
c. $\hat{T} = 126{,}5°$ (opp. $\angle s$ of cyclic quad.)

d. $\hat{A}_2 = 17{,}5°$ (ext. $\angle$ of Δ)
$\therefore \hat{H}_2 = 17{,}5°$ (alternate $\angle s$; $AF \parallel EH$)

3. $NL = 6$ units, $RE = x$ units, $RD = \frac{2}{3}x$ units

$EY = 9$ units and $DL = x + 3$ units.

S is a point on YL and $ED \parallel YL$.



a. Show that L is the midpoint of DN by first solving for x, giving reasons.

$\frac{x}{9} = \frac{\frac{2}{3}x}{x+3}$ (proportional intercept, $ED \parallel YL$)

$\therefore x^2 + 3x = 6x$

$\therefore x^2 - 3x = 0$

$\therefore x(x - 3) = 0$

$\therefore x = 0$ (impossible) or $x = 3$

if $x = 3$

so $DL = LN = 6$ and L is midpoint of DN

b. If SL = 1,4 units then write down the length of DE.

2,8 units

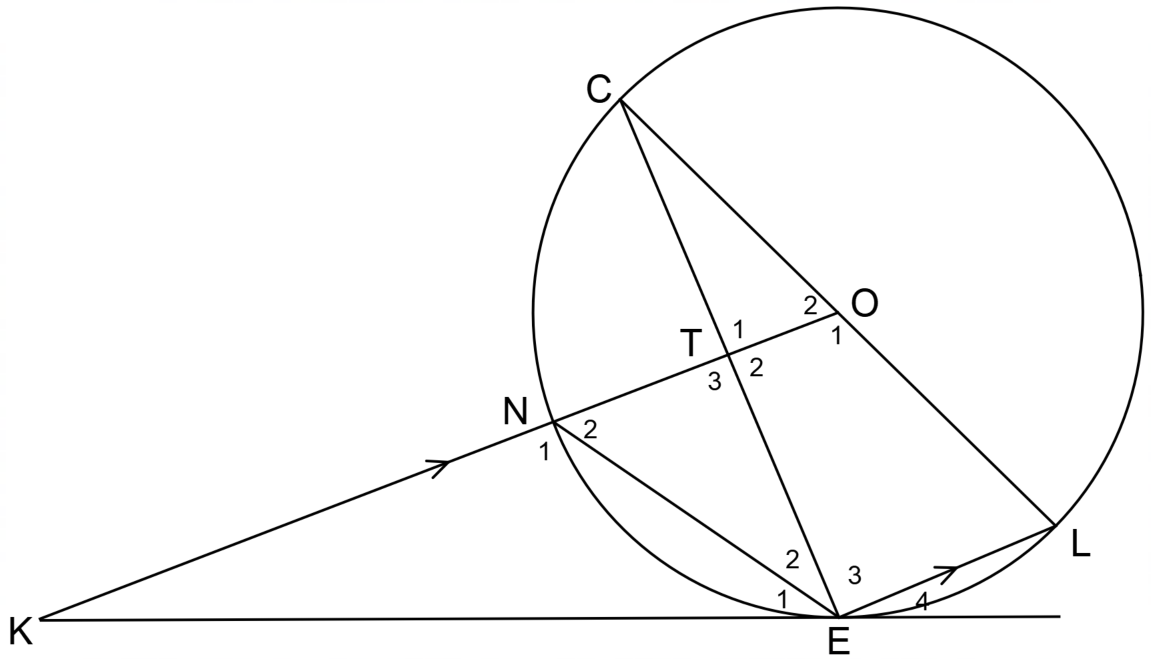
c. If area $\Delta RED = 2{,}7u^2$ find the area of ΔREN

$\frac{\Delta REN}{\Delta RED} = \frac{14}{2}$ (shared heights)

$\therefore \Delta REN = 7 \times 2{,}7$

$\therefore \Delta REN = 18{,}9u^2$

4. In the diagram O is the centre of the circle CNEL. KE is a tangent at E. $KO \parallel EL$



a. Prove that CT = TE.

$\hat{E}_3 = 90°$ ($\angle$ in semi-circle)

$\therefore \hat{T}_1 = 90°$ (corr. $\angle s$ with $KO \parallel EL$)

$\therefore CT = TE$ ($\perp$ from centre)

OR

$\frac{CO}{OL} = \frac{CT}{TE}$ (proportional intercept $KO \parallel EL$)

but $CO = OL$ (radii)

$\therefore \frac{CT}{TE} = 1$ or $CT = TE$

b. Prove that COEK is a cyclic quadrilateral.

$\hat{E}_4 = \hat{K}$ (corr. $\angle s$ with $KO \parallel EL$)

but $\hat{E}_4 = \hat{C}$ (tan chord)

$\therefore \hat{E}_4 = \hat{K}$

$\therefore COEK$ is cyclic (conv. $\angle s$ same segment)

c. Prove that $\Delta COT \parallel\!| \Delta KET$

$\hat{C} = \hat{K}$ (proved)

$\hat{T}_1 = \hat{T}_3$ (vert. opp. $\angle s$)

$\therefore T\hat{E}K = \hat{O}_2$ (3^{rd} $\angle$ of Δ)

$\therefore \Delta COT \parallel\!| \Delta KET$ (AAA)

d. Find CO if OT = 1 and KT = 9 unties.

$\frac{CT}{KT} = \frac{OT}{ET}$ $(\parallel\!| \Delta s)$

$\therefore CT \times ET = OT \times KT$

but $CT = ET$ (proved)

$\therefore CT^2 = 1 \times 9$

$\therefore CT = 3$

Now $CO^2 = 1^2 + 3^2 = \sqrt{10}$ (Pythagoras)

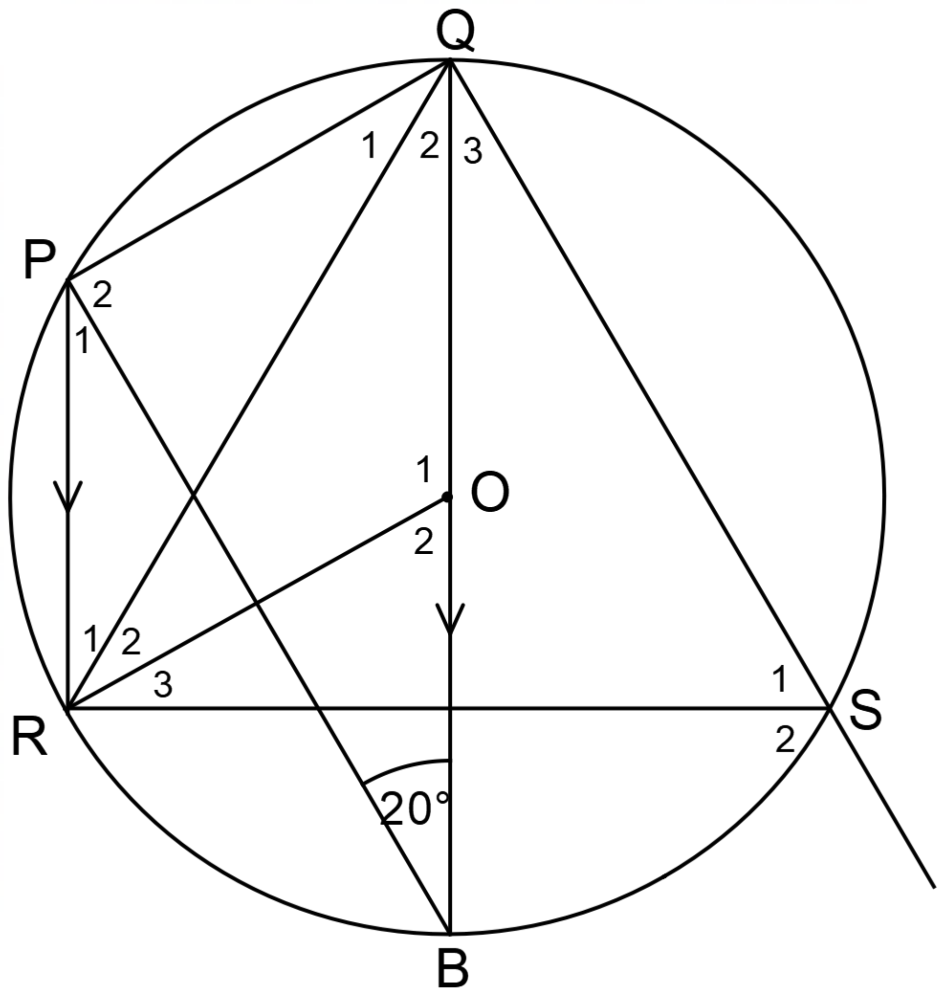
e. Find the size of $\hat{K}$ to 2 d.p.

$\tan\left(\hat{K}\right) = \frac{3}{9}$

$\therefore \hat{K} = \tan^{-1}\left(\frac{1}{3}\right)$

$\therefore \hat{K} = 18{,}43°$

5. O is the centre, $PR \parallel QB$. QO is a diameter. $\hat{B} = 20°$



a. Find, $\hat{P}_1$; $\hat{Q}_2$ and $\hat{O}_2$ with reasons

$\hat{P}_1 = 20°$ (alternate $\angle s$ $PR \parallel QB$)

$\hat{O}_2 = 40°$ ($\angle$ at centre)

$\hat{Q}_2 = 20°$ (ext. $\angle$ of isos. Δ, radii)

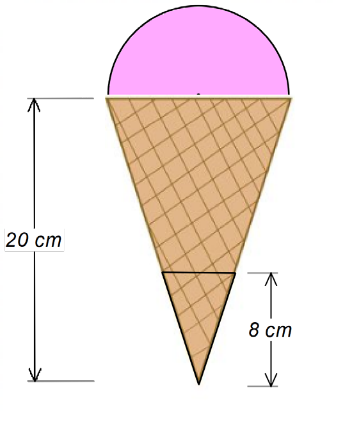
b. Calculate, with reasons, $\hat{S}_2$

$\hat{P}_2 = 90°$ ($\angle$ in semi-circle)

$\therefore \hat{P} = 110°$

$\therefore \hat{S}_2 = 110°$ (ext. $\angle$ of cyclic quad)

6. An ice cream cone is as shown in the picture. It is made up of a cone and a hemisphere with measurements as shown. The bottom 8 cm of the cone are filled with chocolate with a volume of 24 π cm^3.



Determine the overall volume of the cone and the ice cream. You can assume that the thickness of the cone is negligible.

volume of cone $= \frac{1}{3}\pi r^2 ht$ and volume of sphere $= \frac{4}{3}\pi r^3$

radius of chocolate section $= r$

$\frac{1}{3}\pi r^2 \times 8 = 24\pi$

$\therefore r^2 = 9$

$\therefore r = 3\text{cm}$

let radius of whole cone $= R$

by similar triangles

$\frac{R}{20} = \frac{3}{8}$

$R = 7{,}5$ cm

$V = \frac{1}{2}\left(\frac{4}{3} \times \pi \times 7{,}5^3\right) + \frac{1}{3}(\pi \times 7{,}5^2 \times 20)$

$V = 2\,061{,}7 \text{ cm}^3$

a BIG ice cream!