

Grade 12 Mathematics

Paper 2 Revision Workshop

17th October 2026



NAME: WORKED SOLUTIONS



**SHAYA IZIBALO
MATHS INSTITUTE**

in
partnership
with



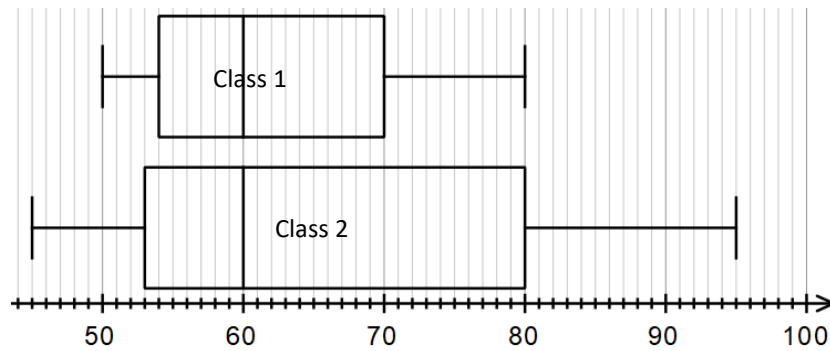
KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

Supported
by



QUESTION 1

(a) The marks on a Maths test for two different classes are displayed in the box and whisker plots below:



- (i) Determine the median mark for class 1. (1)

60

- (ii) Determine the inter-quartile range for class 2. (2)

$$80 - 53 = 27$$

- (iii) Describe the skewness / symmetry of the data in both classes, justifying your answer. (2)

Both are skewed right / positively skewed since the data above the median is more spread out than that below it.

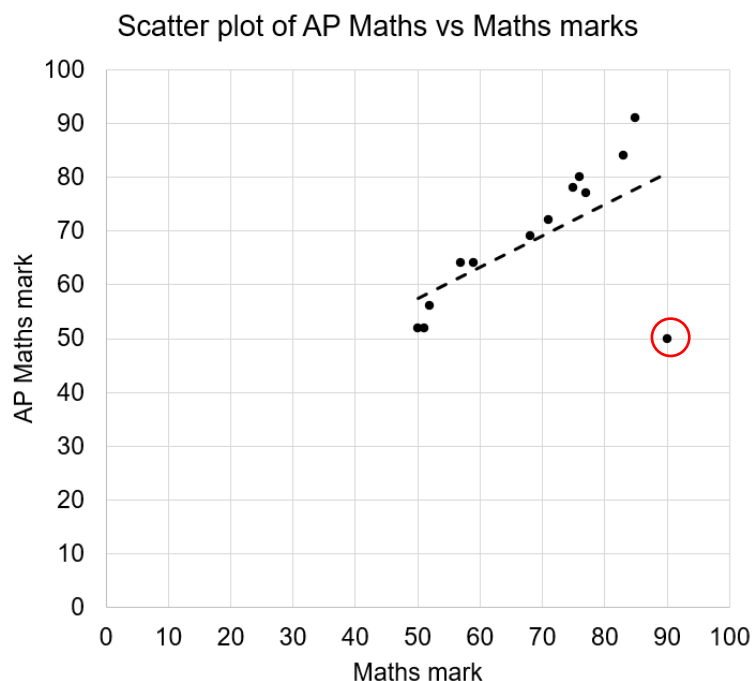
- (iv) Give p if 75% of the candidates in class 1 achieved a mark of greater than p . (1)

53

- (v) Which class has the greater standard deviation? Justify your answer. (2)

Class 2 since the data is more spread out.

- (b) The Maths and AP Maths marks of 13 students are plotted below:



- (i) Comment on the nature and strength of the relationship between the performance in the two subjects. (2)

There is a reasonably strong, positive relationship. Students who do better Maths are likely to do better on AP Maths.

- (ii) The teacher plans to use the equation of the least squares regression line (drawn with a dashed line) to predict the AP Maths score of a student who scored 30% for Maths. Comment on the possible reliability of this. (2)

Despite the reasonably strong correlation, this is potentially risky as extrapolation is involved

- (iii) One of the students could be considered an outlier. Circle this data point. (1)

- (iv) Suppose the outlier is removed from the data set. How would this affect:

- (1) the correlation coefficient? (1)

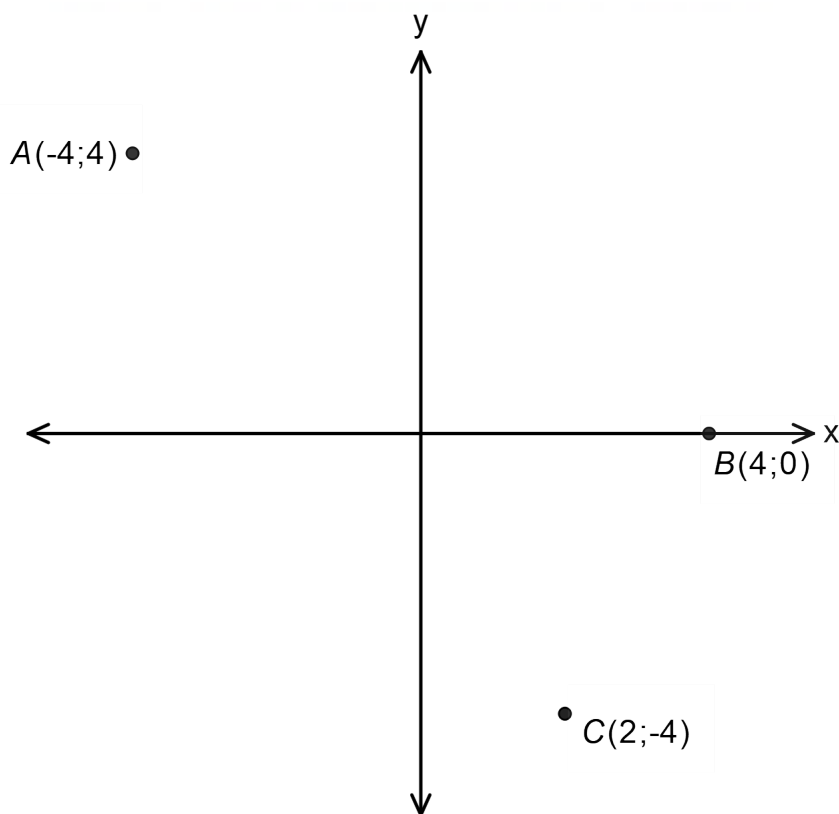
It will strengthen, moving closer to 1.

- (2) the gradient of the least squares regression line? (1)

The gradient will increase.

QUESTION 2

(a) Consider the points as plotted below:



- (i) Show that $\hat{ABC} = 90^\circ$ (3)

$$mAB = -\frac{4}{8} = -\frac{1}{2}$$

$$mBC = \frac{4}{2} = 2$$

$$\text{since } mAB \times mBC = -1 \quad \hat{ABC} = 90^\circ$$

Can also use lengths and converse of Pythagoras

- (ii) Hence, or otherwise, determine the area of $\triangle ABC$ (4)

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \text{base} \times \text{ht} \\ &= \frac{1}{2} \times AB \times BC \\ &= \frac{1}{2} \times \sqrt{(-4 - 4)^2 + (4)^2} \times \sqrt{(4 - 2)^2 + (0 + 4)^2} \\ &= 20 \text{ units}^2 \end{aligned}$$

- (iii) Find the mid-point of line BC (2)



$$\text{mid-pt of } BC = \left(\frac{2+4}{2} ; \frac{0+ -4}{2} \right)$$

$$\text{mid-pt of } BC = (3 ; -2)$$

- (iv) Find point D if ABCD (in that order) is a rectangle. (2)

As we move from B to C, x decreases by 2 and y by 4
The same must happen when we move from A to D. So,

$$D(-6;0)$$

- (v) Find the equation of AB. (3)

$$m_{AB} = -\frac{1}{2}$$

$$y - 4 = -\frac{1}{2}(x + 4)$$

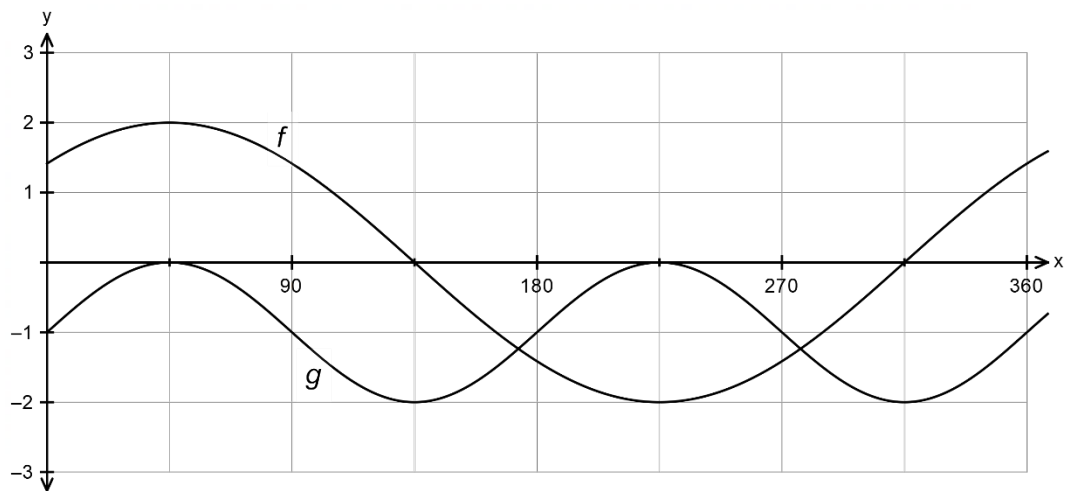
$$y = -\frac{1}{2}x + 2$$

- (b) Give the equation of a circle with centre (5;0) and with the y-axis as a tangent. (3)

$$(x - 5)^2 + y^2 = 25$$

QUESTION 3

- (a) Give the equations of the functions f and g drawn below: (4)



$$f(x) = 2\cos(x - 45^\circ) \text{ or } 2\sin(x + 45^\circ) \text{ and } g(x) = \sin(2x) - 1$$

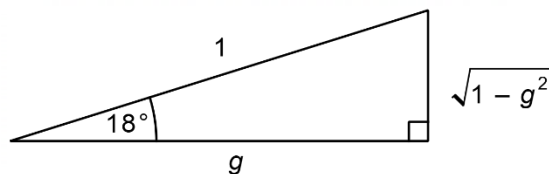
- (b) Suppose that $\cos 18^\circ = g$. Determine the following in terms of g :

(i) $\cos 342^\circ$ (1)
 $= \cos 18^\circ$
 $= g$

(ii) $\sin(-108^\circ)$ (2)
 $= -\cos 18^\circ$
 $= -g$

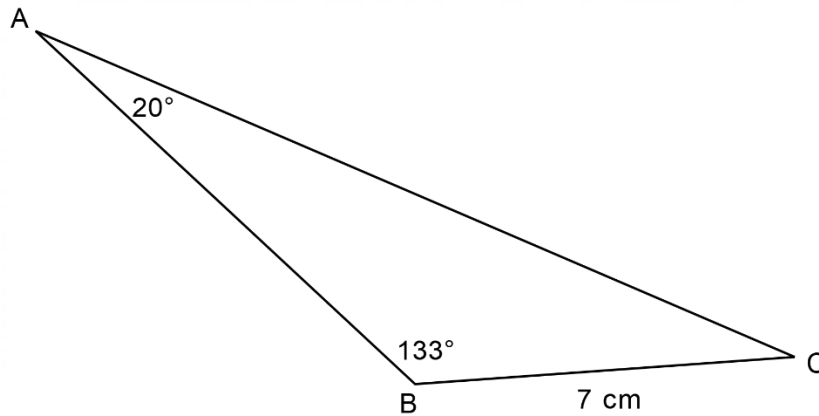
(iii) $\cos 144^\circ$ (3)
 $= \cos(180^\circ - 36^\circ)$
 $= -\cos 36^\circ$
 $= -(2\cos^2 18^\circ - 1)$
 $= 1 - 2g^2$

(iv) $\tan 198^\circ$ (3)
 $= \tan 18^\circ$
 $= \frac{\sqrt{1 - g^2}}{g}$



(c) Determine the area of $\triangle ABC$ to one decimal place.

(5)



$$\hat{C} = 27^\circ$$

$$\frac{AB}{\sin 27^\circ} = \frac{7}{\sin 20^\circ}$$

$$AB = \frac{7(\sin 27^\circ)}{\sin 20^\circ}$$

$$AB = 9.3 \text{ cm}$$

$$\text{Area } \triangle ABC = \frac{1}{2}(9.29)(7)\sin(133^\circ)$$

$$\text{Area } \triangle ABC = 23.8 \text{ cm}^2$$

(d) Solve $\sin(2\theta + 10^\circ) = -\frac{1}{2}$ for $\theta \in [-180^\circ; 180^\circ]$

(6)

key $\angle = 30^\circ$

quadrants 3 and 4

$$\therefore 2\theta + 10^\circ = 210^\circ + 360k \quad \text{or} \quad 2\theta + 10^\circ = 330^\circ + 360k \quad \text{both with } k \in \mathbb{Z}$$

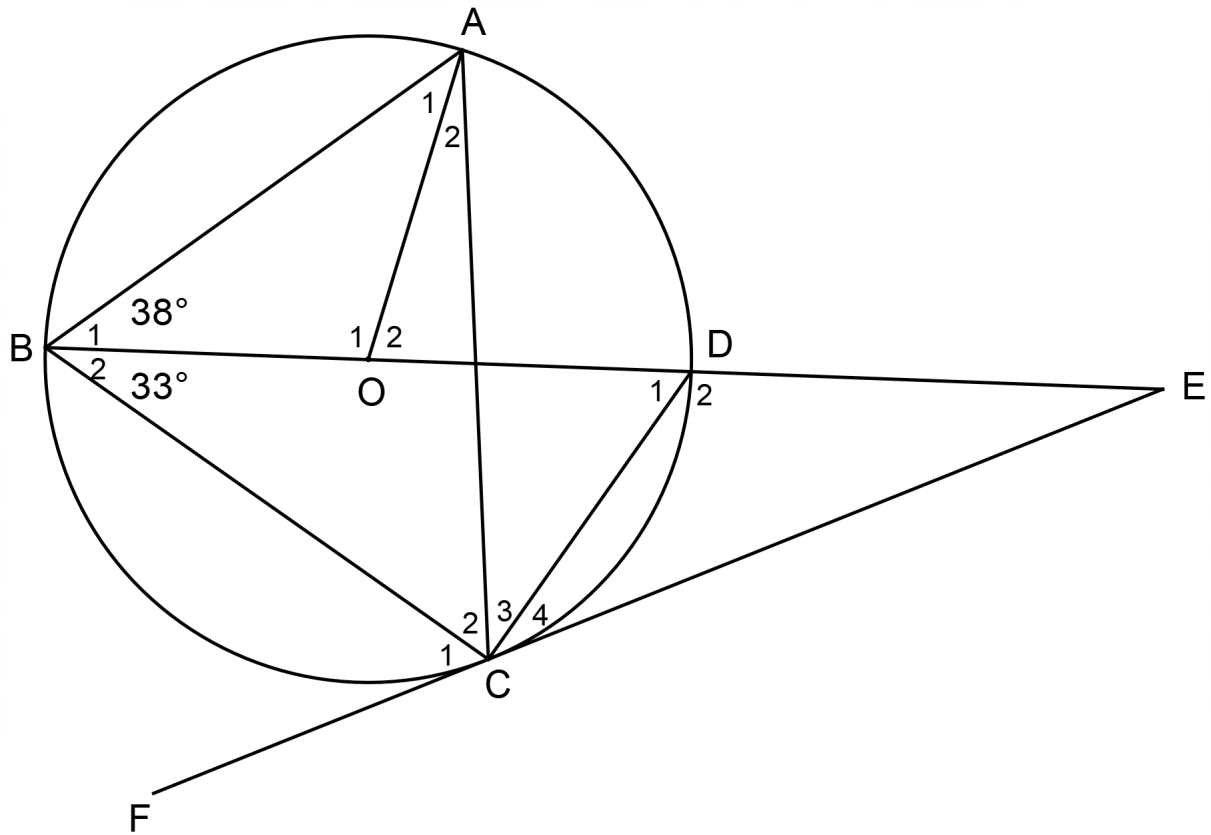
$$\therefore \theta = 100^\circ + 180k \quad \text{or} \quad \theta = 160^\circ + 180k$$

$$\therefore \theta = -80^\circ \text{ or } 100^\circ \text{ or } -20^\circ \text{ or } 160^\circ$$



QUESTION 4

- (a) In the diagram below BOD is a diameter of the circle, centre O. BD is extended to E and FCE is a tangent to the circle at C. $\hat{B}_1 = 38^\circ$ and $\hat{B}_2 = 33^\circ$.



Determine, with reasons, the magnitude of:

- (i) $\hat{BCD} = 90^\circ$ ($\angle$ in semi-circle) (2)

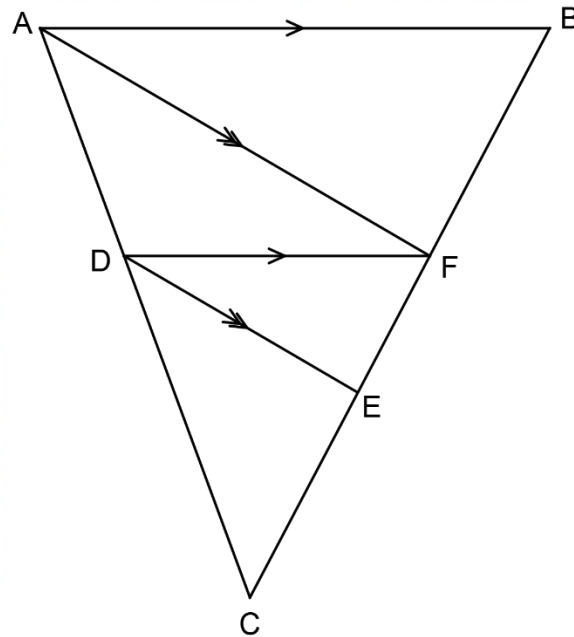
(ii) $\hat{A}_1 = 38^\circ$ (isos. Δ radii) *OR* ($\angle$ s opp = sides radii) (2)

(iii) $\hat{O}_2 = 76^\circ$ (ext. $\angle$ of Δ) *OR* ($\angle$ at centre) (2)

(iv) $\hat{C}_4 = 33^\circ$ (tan chord) (2)



- (b) In the diagram below $AB \parallel DF$ and $AF \parallel DE$. $CF : FB = 3 : 2$



Determine, with reasons:

(i) $AD : DC = 2 : 3$ (prop. int. thm) OR (line $\parallel$ to one side) (2)

(ii) area $\triangle AFD$: area $\triangle DFC = 2 : 3$ (shared heights) (2)

(iii) $EF : FB$ (4)

$CF : FB = 3 : 2$ (given) so let $CF = 3k$ and $FB = 2k$

now $CE : EF = CD : DA = 3 : 2$ (prop. int. thm) OR (line $\parallel$ one side)

so, let $CE = 3g$ and $EF = 2g$

now $CF = 5g = 3k$

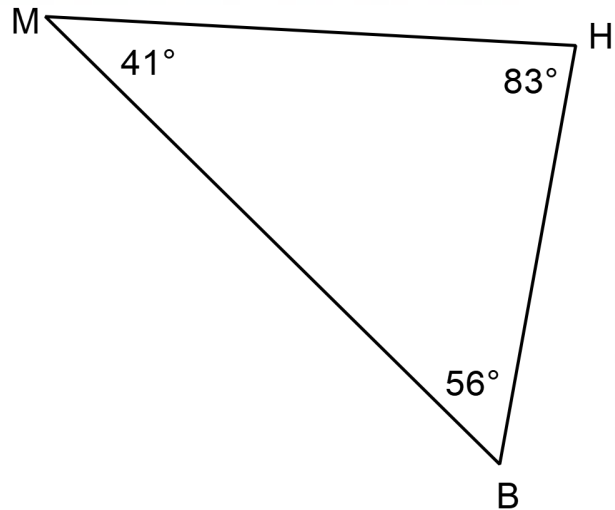
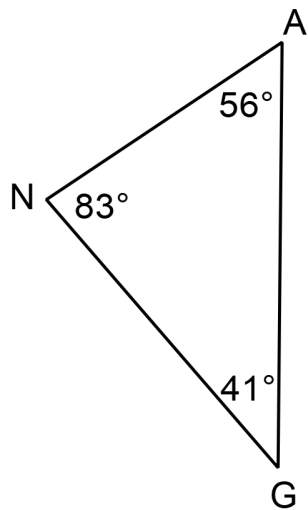
so $k = \frac{5g}{3}$ and $FB = \frac{10g}{3}$

so $EF : FB = \frac{2g}{\frac{10g}{3}} = \frac{3}{5}$

- (c) In the diagram below the bigger triangle is 2,25 times bigger in area than the

Smaller one. Determine $\frac{AN}{BH}$ giving reasons.

(3)



Since the triangles are similar, their areas are in a ratio equal to the square of the ratio of their sides.

$$\text{So, } \frac{AN}{HB} = \sqrt{\frac{1}{2.25}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

QUESTION 5

(a) Consider the numbers $2p$; $2p$; $6p$; $10p$

(i) Determine the standard deviation in terms of p (4)

$$\text{mean} = 5p$$

$$\sigma^2 = \frac{(2p - 5p)^2 + (2p - 5p)^2 + (6p - 5p)^2 + (10p - 5p)^2}{4}$$

$$\sigma^2 = \frac{9p^2 + 9p^2 + p^2 + 25p^2}{4}$$

$$\sigma^2 = \frac{44p^2}{4} = 11p^2$$

$$\sigma = \sqrt{11}p$$

(ii) Hence or otherwise determine the standard deviation in terms of p of the following set of numbers:

$3p$; $3p$; $7p$; $11p$ (2)

Since a fixed amount is being added to each data item the standard deviation will be unchanged at $\sqrt{11}p$

QUESTION 6

(a) Consider the circle $x^2 + y^2 + 6x - 10y = 31$



- (i) Determine the equation of the tangent at the point $A(4;1)$ (6)

$$(x+3)^2 + (y-5)^2 = 31 + 25 + 9 = 65$$

centre is $(-3;5)$ and $r = \sqrt{65}$

$$\text{so, } m \text{ of radius} = \frac{5-1}{-3-4} = -\frac{4}{7}$$

$$\text{so, } m \text{ of tangent} = \frac{7}{4} \text{ (rad } \perp \text{ tan)}$$

$$y-1 = \frac{7}{4}(x-4)$$

$$y = \frac{7}{4}x - 6$$

- (ii) Suppose a tangent (not the one you found above) is drawn from $B(12;-4)$ to a point of contact with the circle, D. Determine the length of BD. (4)

Suppose O is the centre

$$\text{then } BO^2 = 15^2 + 9^2 = 306$$

suppose the length of the tangent is d

$$\text{then } d^2 + r^2 = BO^2 \text{ (Pythagoras)}$$

$$\therefore d^2 + 65 = 306$$

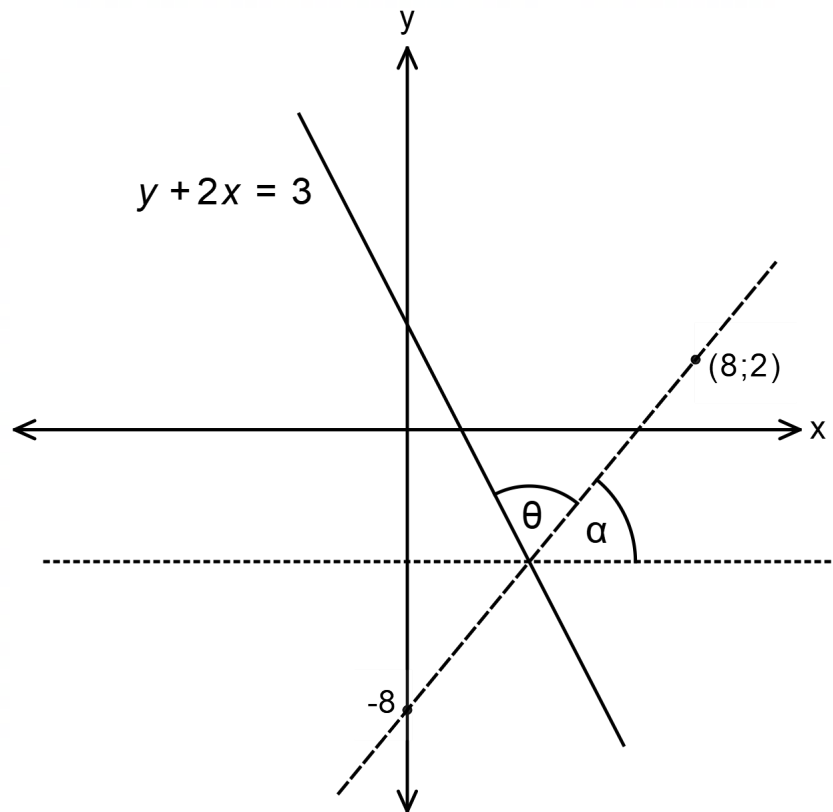
$$\therefore d^2 = 241$$

$$\therefore d = \sqrt{241}$$

$$\therefore d = 15.52 \text{ units}$$



(b) Consider the diagram below.



(i) Determine the gradient of the dashed line. (2)

$$m = \frac{10}{8} = \frac{5}{4}$$

(ii) Hence, or otherwise, determine the size of θ (5)

$$\angle \text{ of inclination of } y + 2x = 3 = 180^\circ - \tan^{-1} 2 = 116.6$$

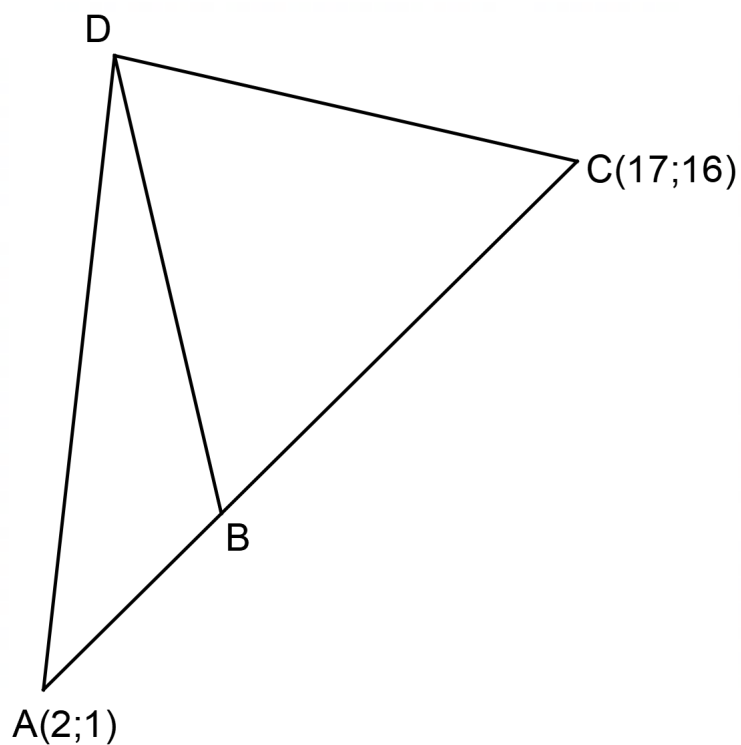
$$\angle \text{ of inclination of dashed line} = \tan^{-1} \left(\frac{5}{4} \right) = 51.34$$

$$\therefore \theta = 116.6^\circ - 51.3^\circ$$

$$\therefore \theta = 65.3^\circ$$

- (c) Determine the coordinates of point B if $\frac{\Delta ABD}{\Delta DBC} = \frac{1}{2}$. Give reasons for your answer.

(5)



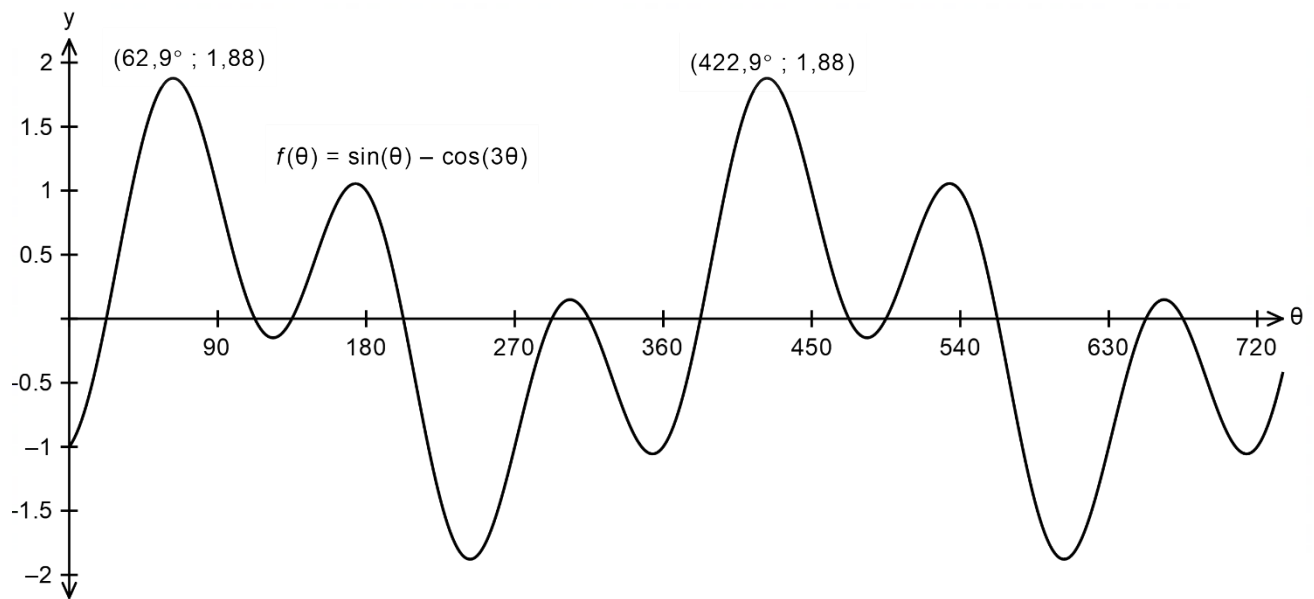
Since the triangles share a height their bases will be in the same ratio as their areas
So, point B is one-third of the way between A and C

$$B\left(2 + \frac{1}{3} \times 15 ; 1 + \frac{1}{3} \times 15\right)$$

$$= B(7 ; 6)$$

QUESTION 7

(a) The function $f(\theta) = \sin(\theta) - \cos(3\theta)$ is drawn below for $\theta \in [0^\circ; 720^\circ]$



(i) Give the period of f (2)

360°

(ii) Determine a general solution for the x-intercepts of f . (6)

$$\sin(\theta) - \cos(3\theta) = 0$$

$$\sin(\theta) = \cos(3\theta)$$

$$\sin(\theta) = \sin(90^\circ - 3\theta)$$

$$\theta = 90^\circ - 3\theta + 360k \quad \text{or} \quad \theta = 180^\circ - (90^\circ - 3\theta) + 360k \quad \text{both with } k \in \mathbb{Z}$$

$$\theta = 22.5^\circ + 90k \quad \text{or} \quad \theta = -45^\circ + 180k$$



(b) Consider the identity: $\frac{\sin\theta}{\cos\theta - \sin\theta} + \frac{\sin\theta}{\cos\theta + \sin\theta} = \tan 2\theta$

(i) Prove it! (5)

$$\begin{aligned} LHS &= \frac{\sin\theta}{\cos\theta - \sin\theta} + \frac{\sin\theta}{\cos\theta + \sin\theta} \\ &= \frac{\sin\theta(\cos\theta + \sin\theta) + \sin\theta(\cos\theta - \sin\theta)}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)} \\ &= \frac{\sin\theta\cos\theta + \sin^2\theta + \sin\theta\cos\theta - \sin^2\theta}{\cos^2\theta - \sin^2\theta} \\ &= \frac{2\sin\theta\cos\theta}{\cos 2\theta} \\ &= \frac{\sin 2\theta}{\cos 2\theta} \\ &= \tan 2\theta \\ &= RHS \end{aligned}$$

(ii) Hence, or otherwise solve $\frac{\sin\theta}{\cos\theta - \sin\theta} + \frac{\sin\theta}{\cos\theta + \sin\theta} = -1$
for $\theta \in [-180^\circ; 180^\circ]$ (4)

$$\tan 2\theta = -1$$

$$\text{key } \angle = 45^\circ$$

quadrants 2 and 4

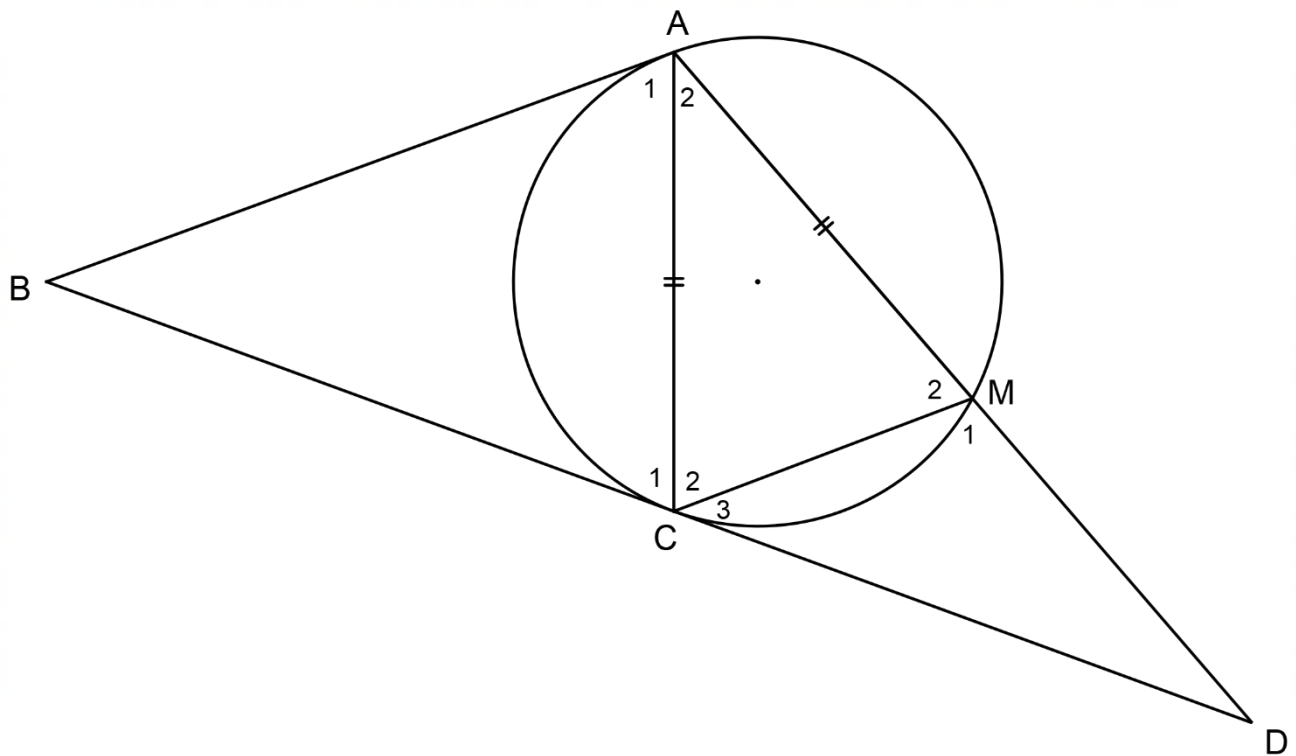
$$2\theta = 135^\circ + 180k \text{ with } k \in \mathbb{Z}$$

$$\theta = 67.5^\circ + 90k$$

$$\theta = 67.5^\circ \text{ or } 157.5^\circ \text{ or } -22.5^\circ \text{ or } -112.5^\circ$$

QUESTION 8

- (a) AB and BC are tangents. AC = AM.



- (i) Prove that $CD^2 = DM \cdot AD$

(5)

$$\begin{aligned}
 \hat{C}_3 &= \hat{A}_2 \text{ (tan chord)} \\
 \hat{D} &\text{ common} \\
 \hat{M}_1 &= \hat{C}_2 + \hat{C}_3 \text{ (}\angle\text{s of } \Delta\text{)} \\
 \therefore \Delta CMD &\parallel\parallel \Delta ACD \text{ (AAA)} \\
 \therefore \frac{CD}{AD} &= \frac{MD}{CD} \text{ (}\parallel\parallel \Delta\text{s)} \\
 \therefore CD^2 &= AD \cdot MD
 \end{aligned}$$

(ii) Prove that $\frac{DM}{MA} = \frac{DC}{CB}$ (4)

$$\hat{A}_1 = \hat{M}_2 \text{ (tan chord)}$$

$$\text{but } \hat{M}_2 = \hat{C}_2 \text{ (}\angle\text{s opp = sides) OR (isos. } \Delta\text{)}$$

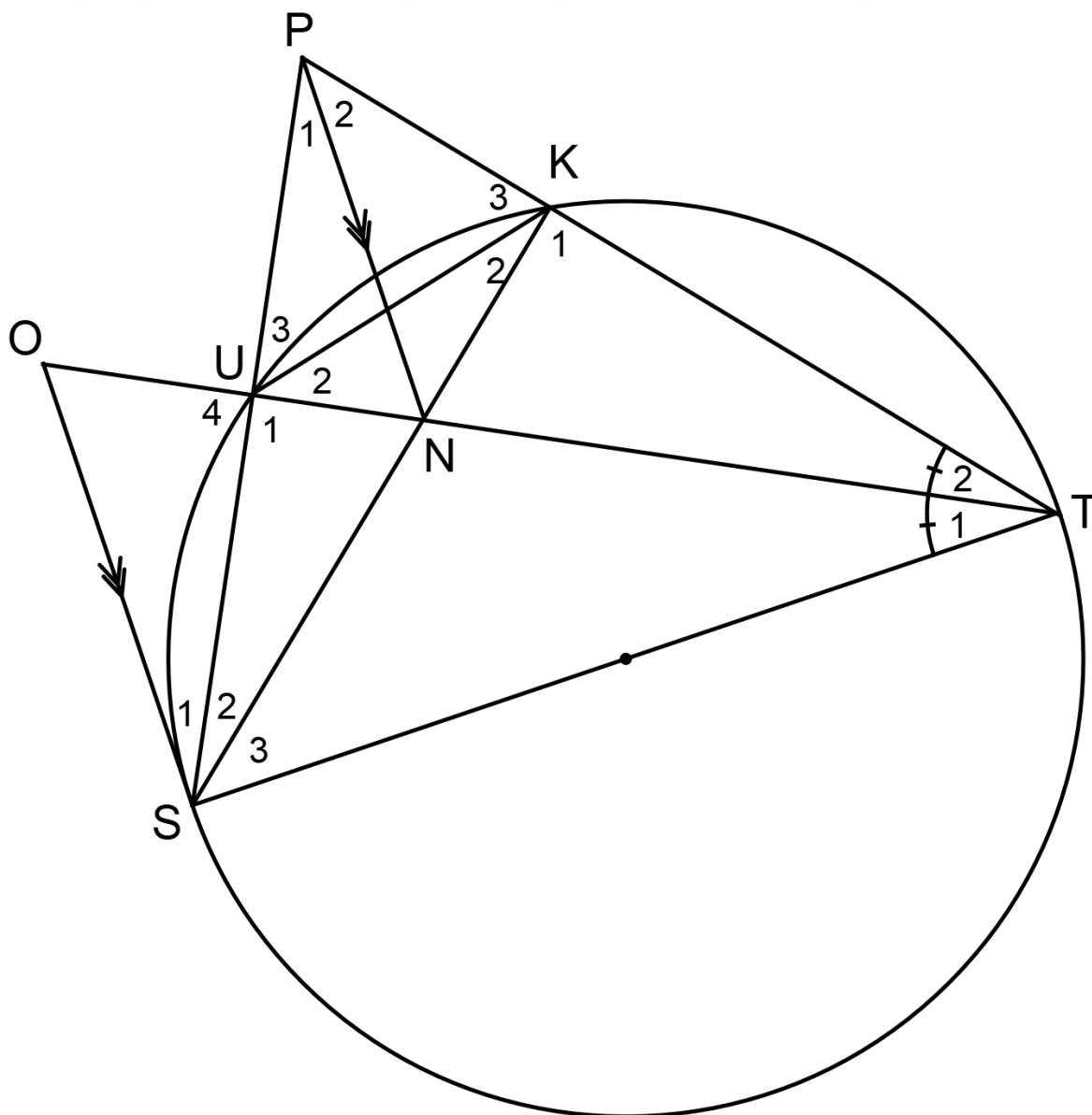
$$\therefore \hat{A}_1 = \hat{C}_2$$

$$\therefore CM \parallel AB \text{ (alt. } \angle\text{s =)}$$

$$\therefore \frac{DM}{MA} = \frac{DC}{CB} \text{ (line } \parallel \text{ to one side of } \Delta\text{) OR (prop. int. thm)}$$



(b) In the diagram below, ST is the diameter of the circle. $OS \parallel PN$.



- (i) Prove that PUNK is a cyclic quadrilateral. (5)

$$\hat{K}_1 = 90^\circ \text{ (}\angle \text{ in semi-circle)}$$

$$\hat{U}_1 = 90^\circ \text{ (}\angle \text{ in semi-circle)}$$

$$\therefore \hat{U}_2 + \hat{U}_3 = 90^\circ \text{ (}\angle \text{s on str. line)}$$

$$\therefore \hat{K}_1 = \hat{U}_2 + \hat{U}_3$$

$\therefore$ PUNK is cyclic (conv. ext. $\angle$ of cyclic quad)

- (ii) Prove that SO is a tangent to the given circle (KUST) (4)

$$\hat{S}_1 = \hat{P}_1 \text{ (alt. } \angle \text{s on } \parallel \text{ lines)}$$

$$\text{but } \hat{P}_1 = \hat{K}_2 \text{ (}\angle \text{s in same segment)}$$

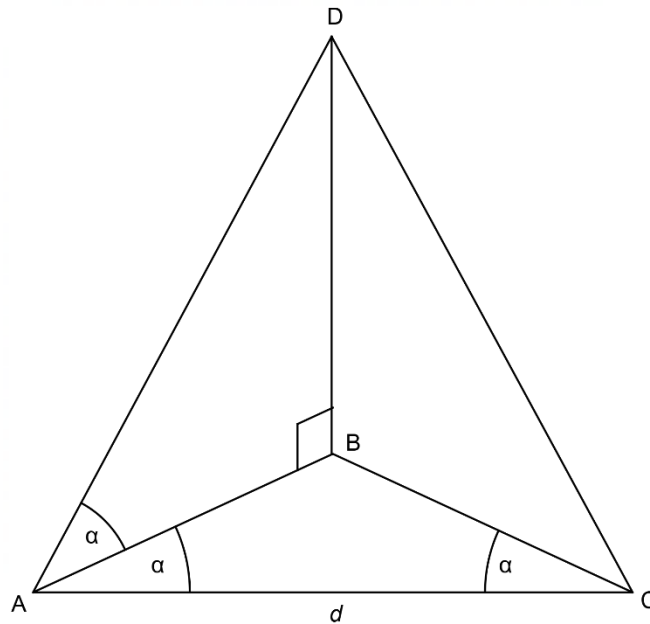
$$\text{and } \hat{K}_2 = \hat{T}_1 \text{ (}\angle \text{s in same segment)}$$

$$\therefore \hat{S}_1 = \hat{T}_1$$

$\therefore$ SO is a tangent (conv. tan chord)

QUESTION 9

- (a) A vertical tower DB has its foot B in the same horizontal plane as points A and C. A and C are d metres apart. $\hat{BAC} = \hat{BCA} = \alpha$ and the angle of elevation of D from A is α .



Prove that the height of the tower is given by $DB = \frac{d \sin \alpha}{2 \cos^2 \alpha}$ (6)

$$\frac{AB}{\sin \alpha} = \frac{d}{\sin(180^\circ - 2\alpha)}$$

$$AB = \frac{d \sin \alpha}{\sin 2\alpha}$$

$$AB = \frac{d \sin \alpha}{2 \sin \alpha \cos \alpha}$$

$$AB = \frac{d}{2 \cos \alpha}$$

$$\tan \alpha = \frac{DB}{AB}$$

$$\tan \alpha = \frac{DB}{AB}$$

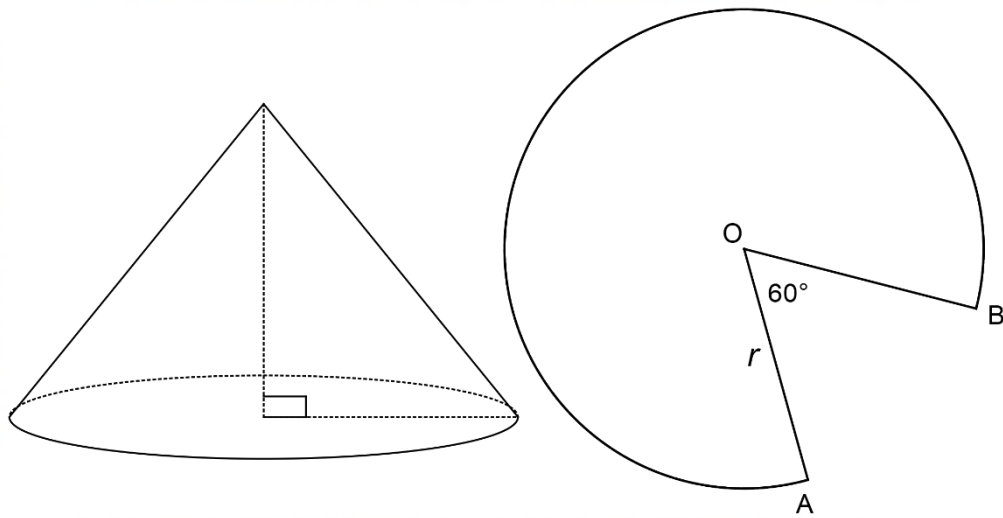
$$DB = AB \tan \alpha$$

$$DB = \frac{d}{2 \cos \alpha} \times \frac{\sin \alpha}{\cos \alpha}$$

$$DB = \frac{d \sin \alpha}{2 \cos^2 \alpha}$$



- (b) A right circular cone as shown is made by cutting a sector out of a circle of radius, r and then taping edge OA and OB together. Determine the size of the angle which the sides make with the base. (6)



$$\begin{aligned} \text{the circumference of the circular base} &= \frac{300}{360} \times 2\pi r \\ &= \frac{5}{3}\pi r \end{aligned}$$

$$\text{now the diameter of the circular base is } \frac{c}{\pi} = \frac{\frac{5}{3}\pi r}{\pi} = \frac{5}{3}r$$

$$\text{so, the radius of the circular base} = \frac{5}{6}r$$

$$\text{the slant height} = r$$

if θ is angle the side makes with the base then

$$\cos\theta = \frac{\frac{5}{6}r}{r} = \frac{5}{6}$$

$$\theta = \cos^{-1}\left(\frac{5}{6}\right)$$

$$\theta = 33.56^\circ$$

