

Expressing rational numbers as a ratio (fraction) of two whole numbers

Remember we said that any rational number can be written as a ratio of two whole numbers? Well let's see how to do this by considering some examples.

If the number terminates (ends) then we can do the following:

$$\text{if } x = 0,375$$

$$\text{then } 1\,000x = 375$$

$$\text{and } x = \frac{375}{1\,000}$$

$$\text{if } y = 2,87$$

$$\text{then } 100y = 287$$

$$\text{and } y = \frac{287}{100}$$

Note that the above fractions may be able to be simplified but we have already achieved our goal

Consider these ones which don't terminate but which repeat:

$$\text{if } x = 0,55\,555\dots$$

$$\text{then } 10x = 5,55\,555\dots$$

subtracting the top equation from the bottom equation gives:

$$9x = 5$$

$$\text{so, } x = \frac{5}{9}$$

$$\text{if } x = 2,393\,939\overline{39}$$

$$\text{then } 100x = 239,393\,939\overline{39}$$

subtracting the top equation from the bottom equation gives:

$$99x = 237$$

$$\text{so, } x = \frac{237}{99}$$

You try these:

$$3,572$$

$$2,345\,345\overline{345}$$

$$1,687$$

$$0,686\,868\overline{68}$$

$$1,555\overline{5}$$

$$0,00\,045$$

Estimation and calculation tricks

Suppose we want to multiply 199 by 21.

This will be similar in size to 200×20 which is 4000. By working out the exact answer work out the **error**.

Instead of long multiplication, let us try another way of working out 199×21

$$\begin{aligned} & 199 \text{ lots of } 21 \\ &= 200 \text{ lots of } 21 \text{ minus } 21 \\ &= 200 \times 21 - 21 \\ &= (2 \times 100) \times 21 - 21 \\ &= (2 \times 21) \times 100 - 21 \\ &= 42 \times 100 - 21 \\ &= 4\,200 - 21 \\ &= 4\,179 \end{aligned}$$

Try using this trick to calculate the following:

99×57

101×43

999×67

50×99

27×101

$48 \times 1\,001$

Space for your teacher to make up some extra examples...

Using addition rather than subtraction.

Suppose you buy something for R13,68 and you pay with a R20 note. Instead of subtracting 13,68 from 20 to get your change he / she normally counts back up to R20 to work out what you are due.

2c to make up to 13,70

30c to make that up to 14,00

R6 to make that up to R20

So, the total change due is $R6 + R0,30 + R0,02 = R6,32$. This is often easier than doing the subtraction.

Determine the following using this technique:

$$500 - 297$$

$$350 - 287$$

$$480 - 375$$

$$250 - 137$$



Space for your teacher to make up some extra examples...