

Prime Numbers

The **prime** numbers are a subset of the natural numbers. They are special numbers since they have **exactly two unique (different) factors** – themselves and 1.

For, example:

5 is prime since its only factors are 1 and 5

1 is not prime since it only has one factor

4 is not prime since it has three factors: 1 ; 2 and 4.

Natural numbers which are not prime are said to be **composite** numbers.

1 is **neither** prime nor composite.

There are 25 prime numbers less than 100. Here they are:

2	3	5	7	11
13	17	19	23	29
31	37	41	43	47
53	59	61	67	71
73	79	83	89	97

There are only two prime numbers which are consecutive (one after the other). Which are they?

Why could there not be another pair of consecutive prime numbers?

Twin Primes are two primes which are two apart. The first pair of twin primes is 3 and 5.

Write down all the sets of twin primes which are less than 100.

Do you think that primes become more common or rarer as the numbers get bigger?

Why?

Do you think there is a biggest prime and why?

Euclid, who lived around 300 BC proved that

Big prime numbers are very useful and valuable in encryption (creating codes). For this reason, there is an ongoing search for ever bigger prime numbers. Let's see what the biggest prime number is so far..... https://en.wikipedia.org/wiki/Largest_known_prime_number

If we wrote it down, one digit per second, how long would it take us?

Play this game and see who can get the highest score.

<https://isthisprime.com/game/>

Goldbach's conjecture

Goldbach stated that any even number greater than 2 can be written as the **sum of 2 prime** numbers.

For example: $12 = 5 + 7$

See if you can write each of the following as the sum of 2 prime numbers:
Remember to **ONLY USE PRIME NUMBERS!**

$12 =$

$10 =$

$16 =$

$20 =$

$36 =$

$42 =$

$48 =$

$70 =$

$82 =$

$94 =$

Note that Goldbach's conjecture has never been proved but nobody has ever come up with an even number that cannot be expressed as the sum of two prime numbers. So, it is accepted to be true even though it has not been proved! There's a challenge for you! 😊

Numbers that have no common factors are said to be **relatively prime**. For example, 8 and 21 are relatively prime even though they are not themselves prime numbers.

When we simplify a fraction, we end up with our numerator (the top) and our denominator being relatively prime. For example: $\frac{48}{60} = \frac{4}{5}$ (4 and 5 are relatively prime).

Give three numbers which are relatively prime:

Are 54 and 3000 relatively prime? Why?

Primes, Multiples and Factors

Let's define the following terms:

Prime:

Multiple

Factor:

Make a sentence using the word FACTOR

Make a sentence using the word MULTIPLE

What is the HIGHEST COMMON FACTOR of some numbers?

What is the LOWEST COMMON MULTIPLE of some numbers?

True or false?

- 14 is a factor of 7 _____
- 3 is a factor of 9 _____
- 8 is a multiple of 2 _____
- 12 is a multiple of 24 _____
- 12 is both a factor and a multiple of 12 _____

1. Write down the first three multiples of each of the following:

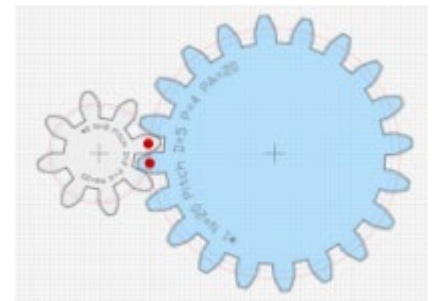
a. 11

b. 7

c. 6

2. Write down **ALL** the factors of each of the following:
- a. 23
 - b. 64
 - c. 48
 - d. 100
 - e. 60
 - f. 36
 - g. What do you notice about the **number** of factors each of the numbers above has?
 - h. When do we get an **odd number** of factors?
 - i. Why?
 - j. Give an example of another number that has an odd number of factors.

3. Write down the first 10 square numbers (perfect squares)
4. Write down the first 6 cubes (perfect cubes)
5. Mr. Khumalo can divide a group of boys in groups of 3, 5 or 8 without having anyone left over. What is the smallest number of boys for which this is possible?
6. Two cogs are turning. The first has 8 teeth, the second 20 teeth. After how many turns of the small cog will they all both be back in their starting positions?



7. Three cogs are turning. The first has 8 teeth, the second 24 teeth and the third has 36 teeth. After how many turns of the small cog will they all be back in their starting positions?

8. Determine the HCF (highest common factor) of the following numbers:

a. 24 and 36

b. 8 and 52

c. 16, 24 and 60

d. 36, 54 and 72