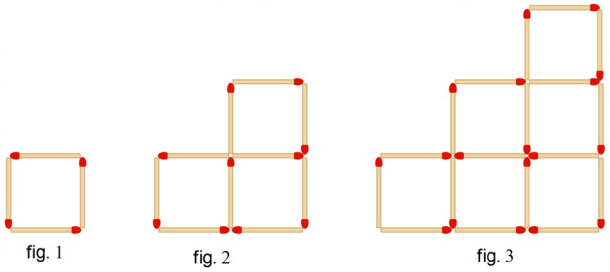
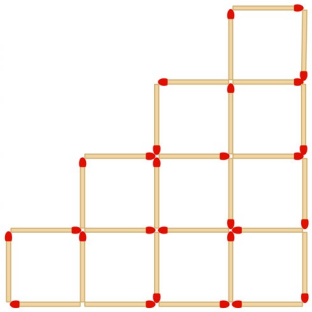
Grade 12 Mathematics

Module 1 – Sequences and Series



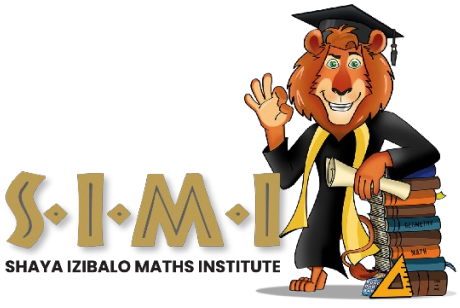


How many matches will be needed to make fig. *n*?

Determine $\sum_{i=1}^{23} (2i+5)$

A frog jumps 2m on his first jump. Every jump thereafter he only jumps three-quarters of the distance of the previous jump. How far does he get if he continues in this manner?

NAME:



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A **sequence** is a set of numbers following some form of pattern or rule. Each number is separated from the next by a semi-colon ;

A **series** is the sum of some numbers following some form of pattern or rule. Each term is separated from the next by a + or – sign.

1 ; 4 ; 7 ; 10 is a **sequence** while _____ is the corresponding **series**

There are three types of sequences we need to be able to deal with:

ARITHMETIC SEQUENCES

These are sequences where we get from one term to the next by **ADDING** a fixed amount (could be negative).

Examples:

2 ; 5 ; 8 ; 11 ; ..... ; ..... ; .....; ..... here we are adding _____

14 ; 10 ; 6 ; 2 ; ..... ; ..... ; ..... ; ..... here we are adding _____

$2x + 12$; $3x + 10$; $4x + 8$; $5x + 6$; ..... ; ..... ; .....; ..here we are adding _____

There are two parameters which affect an arithmetic sequence

The _____ and the_____. d is found by taking any term and

_____

For example: _____

Determine d for the above sequences and write it next to them.

If we know d then we can also get from any term to the next term. Give the next three terms in each of the above sequences.

A very important idea is that:

If three numbers a, b and c are in arithmetic sequence then:

If we know these two parameters, (a and d) then we can write out as many terms of the sequenc as we like.

Let's try. Write down the first five terms of an arithmetic sequence which has:

$a = -5$ and $d = 3$ _____

$a = 12$ and $d = -3$ _____

$a = 4x + 2$ and $d = -x + 2$ _____

We use the notation T_n to denote the n^{th} term of the sequence.

Consider the following sequence:

2 ; 5 ; 8 ; 9 ; 11 ; 17 ; ....

Is it arithmetic? _____

Why? _____

For the above sequence give:

The third term _____

The term which is equal to 5 _____

T_5 = _____

Notice the difference between T_n and n. T_n is the value of the term while n is its position in the sequence.
Notice too that $a = T_1$
Finally, it is important to realise that _____

The formula for the n^{th} term of an arithmetic sequence is _____

We can easily prove this formula and the proof is required for matric.

Let's start off with a specific example and then look at the general case:

Find the n^{th} term of the sequence: $3\,;\,7\,;11\,;15\,;....$

This can simplify to _____

Now let's do the same with the general case:

The formula $T_n = a + (n-1)d$ has four variables in it. In maths, providing we are only missing one variable, we can solve for it. Let us consider some examples:

1. Determine the 17^{th} term of the sequence $7 \;;\; 10 \;;\; 13 \;;\; 16 \;;\; .....$

2. Determine how many terms are in the sequence $35 \;;\; 32 \;;\; 29 \;;\; 26 \;;\; ..... \;;\; -7$

3. Determine the first term of a sequence which has a common difference of 3 and $T_{17} = 55$

4. Determine the common difference of a sequence which has a first term of -4 and $T_{21} = 46$

5. Is 150 a term in the sequence 5 ; 8 ; 11 ; …… Justify your answer with a calculation.

6. How many terms are there in the sequence 502 ; 495 ; 488 ; 481 ; .... ; 103