

# Grade 9 Mathematics

## Module 3: Exponents

Do you  
believe in  
God?

$X^3$

Well, I do  
believe in  
higher powers

$X^5$

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# EXPONENTS LESSON 1

First, we need to become familiar with some terminology:

Suppose we have:  $5^2$

5 is called the base and the 2 is called the exponent.

In Mathematics we use an **exponent** to indicate repeated **multiplication**. Exponents are also sometimes called **powers**.

$a^3$  means  $a \times a \times a$  or 3 lots of  $a$  **multiplying** together.

This must not be confused with:

$3a$  which means  $a + a + a$  or 3 lots of  $a$  adding together.

In this case we call the 3 the coefficient.

Mathematics is a language, and we need to learn to speak it but also to write it.

When we see  $3a$  we say three a but when we see  $a^3$  we say  $a$  cubed or  $a$  to the power of 3.

An important thing to realise is that exponents come before multiplication in our order of operations.

Evaluate (work out the value of) the following:

$$2 \times 5^2 = \underline{2 \times 25 = 50}$$

$$-2^2 = \underline{-(2^2) = -4}$$

## EXERCISE 1

Write the following out in full:

a.  $b^4$

$$= \underline{bbbb}$$

b.  $4g$

$$= \underline{g + g + g + g}$$

c.  $2^3$

$$= \underline{2 \times 2 \times 2}$$

d.  $e^4$

$$= \underline{eeee}$$

e.  $5^6$

$$= \underline{5.5.5.5.5.5}$$

f.  $4e$

$$= \underline{e + e + e + e}$$

g.  $3^2$

$$= \underline{3 \times 3}$$

h.  $c^6$

$$= \underline{cccccc}$$

i.  $5a$

$$= \underline{a + a + a + a + a}$$

Now write these using a coefficient or an exponent:

a.  $c + c + c + c + c$

$= 5c$   
→

b.  $d \times d \times d$

$= d^3$   
→

c.  $5 \times 5 \times 5 \times 5 \times 5$

$= 5^5$   
→

d.  $3 + 3 + 3 + 3 + 3 + 3 + 3$

$= 7 \times 3$   
→

e.  $g \times g \times g \times g \times g \times g \times g$

$= g^7$   
→

f.  $h + h + h + h + h$

$= 5h$   
→

g. 35 lots of  $h$  multiplied together

$= h^{35}$   
→

h.  $(d)(d)(d)$

$= d^3$   
→

i. 7 lots of  $b$  added together

$= 7b$   
→

Next, we are going to look at what happens when we multiply numbers which have exponents but with the same bases.

Calculate the following by first writing them out in full:

$a^3 \times a^4$

$= aaa \times aaaa$

$= a^7$   
→

$b^2 \times b^5$

$= bb \times bbbbb$

$= b^7$   
→

$e^4 \times e^4$

$= eeee \times eeee$

$= e^8$   
→

$2^4 \times 2^3$

$= 2.2.2.2 \times 2.2.2$

$= 2^7$   
→

Note: you may have been tempted to say that  $2^4 \times 2^3 = 4^7$  BUT we have 7 twos multiplying together NOT 7 fours multiplying together!

Let's summarise what we have learnt:

When we multiply numbers with the same bases, we ADD the exponents and keep the base as is. We can

write this with symbols as follows:  $a^b \times a^c = a^{b+c}$   
→


Let's practice our new rule.


Remember that we can use a . to represent multiplication. So,  $a.b = a \times b = ab$ .


Likewise,  $(a)(b)$  means  $a \times b$ .


## EXERCISE 2


Simplify, remembering that numbers can multiply in any order. If you are unsure, then go back to basics and write them out in full.


a.  $a^3 \times a^7$   
 $= a^{10}$   



b.  $b^4 \times b$   
 $= b^5$   



c.  $g^3 \times g^2 \times g^4$   
 $= g^9$   



d.  $2^5 \times 2^9$   
 $= 2^{14}$   



e.  $y^2 \times y^6 \times y$   
 $= y^9$   


f.  $3^5 \times 3^3 \times 3^2$   
 $= 3^{10}$   


g.  $f^5 \times g^3 \times f^6 \times g$   
 $= f^{11} g^4$   


h.  $(a^2 b^4)(a^5 b^3)$   
 $= a^7 b^7$   


i.  $a^{64} \times a^6$   
 $= a^{70}$   


j.  $e^{30} g^{10} \times g^{11} e$   
 $= e^{31} g^{21}$   


Now consider multiplying numbers which involve both coefficients and exponents!

For example,  $2x^3 \times 3x^5$

Remember that we can multiply numbers in any order so this can be re-written as:

$$2 \times 3 \times x^3 \times x^5$$

The 2 and the 3 are “normal” numbers in the sense that they do not have powers on them. As always, we multiply them to get 6. Then we have a total of 8 lots of  $x$  multiplying together. The shorthand for this is  $x^8$

So, our final answer is  $6x^8$ .

Notice that  $2 \times 3^2$  is NOT  $6^2$  !

We cannot multiply the 2 and the 3 since they are not both “normal” numbers without exponents.


$2 \times 3^2$  means  $2 \times 3 \times 3$  which works out to 12, not 36 which is what  $6^2$  is equal to.


Time to practise, remembering that practice makes perfect!


Finally, don't forget your rules of integers!


### EXERCISE 3


Simplify:


a.  $3x^2 \times 2x^5$   
 $= 6x^7$   



b.  $4a^2b^3 \times 2ab^4$   
 $= 8a^3b^7$   



c.  $(2m^2)(3m^4)(10m)$   
 $= 60m^7$   



d.  $3b^3 \cdot 2b \cdot 4b^4$   
 $= 24b^8$   



e.  $2x^2y^3z \times 3xy^4z^2$   
 $= 6x^3y^7z^3$   


f.  $(2a^2b)(-4ab^3)$   
 $= -8a^3b^4$   


g.  $3m^2n \times -2m^4n^2$   
 $= -6m^6n^3$   


h.  $4g^2h \cdot 3g^3h^2$   
 $= 12g^5h^3$   


i.  $2 \times 3^5 \times 5 \times 3^{11}$   
 $= 10 \times 3^{16}$   


j.  $3x^3y \times 2x^2y^4 \times 5xy^6$   
 $30x^6y^{11}$   


# EXPONENTS LESSON 2

In the last lesson we looked at multiplying numbers with exponents.

We established our first rule.....

$$a^b \times a^c = a^{b+c}$$

In words, when we multiply numbers with the same base, we add their exponents.

Now we are going to discover what happens when we divide numbers with exponents.

Firstly, let's remind ourselves how we simplify fractions.

Suppose we have  $\frac{8}{24}$

We can divide the top (numerator) and the bottom (denominator) by 2, giving  $\frac{4}{12}$

this is an equivalent fraction. It is still not in simplest form, we can now divide the top and bottom by 4, giving:

$$\frac{1}{3}$$

This is in simplest form. Note that we could have divided the top and bottom by 8 initially and got straight to the answer. The important thing to remember is that we can **simplify fractions by dividing the top and the bottom by the same number**.

Now let's apply this thinking to numbers involving exponents:

$$\frac{a^7}{a^3}$$

Well, if we expand this then we get:  $\frac{a \times a \times a \times a \times a \times a \times a}{a \times a \times a}$

Now if we divide the top and bottom by  $a$  we get:  $\frac{\cancel{a}^1 \times a \times a \times a \times a \times a \times a}{\cancel{a}^1 \times a \times a}$

If we do it again, we get  $\frac{\cancel{a}^1 \times \cancel{a}^1 \times a \times a \times a \times a \times a}{\cancel{a}^1 \times \cancel{a}^1 \times a}$  and again we get  $\frac{\cancel{a}^1 \times \cancel{a}^1 \times \cancel{a}^1 \times a \times a \times a \times a}{\cancel{a}^1 \times \cancel{a}^1 \times \cancel{a}^1}$

Remember that a 1 remains every time we do the divisions. So, we end up with  $\frac{a^4}{1} = a^4$ .

Now you try these by first expanding them in full:

## EXERCISE 1

a.  $\frac{g^4}{g^2}$

$$= \frac{gggg}{gg}$$

$$= \frac{\cancel{g^1} \cancel{g^1} gg}{\cancel{g^1} \cancel{g^1}}$$

$$= g^2$$

d.  $\frac{2^5}{2^3}$

$$= \frac{2.2.2.2.2}{2.2.2}$$

$$= \frac{\cancel{2^1} \cancel{2^1} \cancel{2^1} .2.2}{\cancel{2^1} \cancel{2^1} \cancel{2^1}}$$

$$= 2^2$$

g.  $h^8 \div h^3$

$$= \frac{hhhhhhhh}{hhh}$$

$$= \frac{\cancel{h^1} \cancel{h^1} \cancel{h^1} hhhh}{\cancel{h^1} \cancel{h^1} \cancel{h^1}}$$

$$= h^5$$

b.  $\frac{b^6}{b^3}$

$$= \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b \cdot b}$$

$$= \frac{\cancel{b^1} \cancel{b^1} \cancel{b^1} bbb}{\cancel{b^1} \cancel{b^1} \cancel{b^1}}$$

$$= b^3$$

e.  $\frac{b^7}{b^2}$

$$= \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b}$$

$$= \frac{\cancel{b^1} \cancel{b^1} \cdot b \cdot b \cdot b \cdot b \cdot b}{\cancel{b^1} \cancel{b^1}}$$

$$= b^5$$

h.  $\frac{u^8}{u^2}$

$$= \frac{uuuuuuuu}{uu}$$

$$= \frac{uuuuuuuu}{uu}$$

$$= \frac{\cancel{u^1} \cancel{u^1} uuuuuu}{\cancel{u^1} \cancel{u^1}}$$

$$= u^6$$

c.  $\frac{h^5}{h}$

$$= \frac{h \cdot h \cdot h \cdot h \cdot h}{h}$$

$$= \frac{\cancel{h^1} \cdot h \cdot h \cdot h \cdot h}{\cancel{h^1}}$$

$$= h^4$$

f.  $\frac{5^7}{5^2}$

$$= \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5}$$

$$= \frac{\cancel{5^1} \cancel{5^1} \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{\cancel{5^1} \cancel{5^1}}$$

$$= 5^5$$

i.  $4^8 \div 4^2$

$$= \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4}$$

$$= \frac{\cancel{4^1} \cancel{4^1} \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{\cancel{4^1} \cancel{4^1}}$$

$$= 4^6$$

When seeing  $\frac{2^5}{2^3}$  you may have been tempted to “divide” the 2s by one another. However, we don’t do that! They are not “normal” numbers without exponents!

We hope by now you may have started to form a rule for dividing numbers with exponents and the same bases?

When we divide numbers with the same bases, we subtract the exponents and keep the base as is.

We can write this with symbols as follows:  $a^b \div a^c = \underline{a^{b-c}}$  or  $\frac{a^b}{a^c} = \underline{a^{b-c}}$

As was the case with multiplication, if we have coefficients, we can divide them or simplify them as normal.

For example:  $\frac{12x^8}{4x^3} = 3x^5$

Time to practise!

## EXERCISE 2

Simplify:

$$\begin{aligned} \text{a. } & \frac{p^{11}}{p^8} \\ & = p^3 \end{aligned}$$

$$\begin{aligned} \text{b. } & \frac{q^{30}}{q^{25}} \\ & = q^5 \end{aligned}$$

$$\begin{aligned} \text{c. } & \frac{2^8}{2^2} \\ & = 2^6 \end{aligned}$$

$$\begin{aligned} \text{d. } & \frac{6x^5}{2x^3} \\ & = 3x^2 \end{aligned}$$

$$\begin{aligned} \text{e. } & \frac{18a^4b^6}{9a^2b^3} \\ & = 2a^2b^3 \end{aligned}$$

$$\begin{aligned} \text{f. } & \frac{15y^8}{-3y^4} \\ & = -5y^4 \end{aligned}$$

$$\begin{aligned} \text{g. } & \frac{12 \times 7^{14}}{6 \times 7^{11}} \\ & = 2 \times 7^3 \end{aligned}$$

$$\begin{aligned} \text{h. } & \frac{a^5b^7c^{19}}{c^{15}b^6a} \\ & = a^4bc^4 \end{aligned}$$

$$\begin{aligned} \text{i. } & \frac{-12g^7h^{12}}{-2g^3h^5} \\ & = 6g^4h^7 \end{aligned}$$

$$\begin{aligned} \text{j. } & \frac{3^7d^4e^9}{3^3d^2e^3} \\ & = 3^4d^2e^6 \end{aligned}$$

Now, consider  $\frac{a^5}{a^5}$ . Well, when we divide two identical quantities, we know that the answer will be 1.

For, example, 23 sweets divided by 23 children is 1 – each child gets 1 sweet.

However, we have also just learnt that when we divide numbers with the same base, we subtract the exponents and keep the base as is.

$$\text{So, } \frac{a^5}{a^5} = a^0 = 1$$

**In fact, any number raised to the power of 0 is 1.** This is a definition rather than a rule but important all the same!

Let's make sure we understand the difference between these:

$$5x^0 \text{ and } (5x)^0$$

$$\begin{aligned} 5x^0 & \qquad \text{but} \qquad (5x)^0 = 1 \\ & = 5 \times 1 \\ & = 5 \end{aligned}$$



Next, we are going to combine our multiplication and division. Now is the time to keep calm! While the sums might look a little bigger, we just tackle them one step at a time.

Consider this example:

$$\begin{aligned} & \frac{2a^3b^4 \times 6ab^5}{3ab^6 \times 2ab^3} \\ &= \frac{12a^4b^9}{6a^2b^9} \\ &= 2a^2b^0 \\ &= 2a^2 \end{aligned}$$

Let's practise some!

### EXERCISE 3

$$\begin{aligned} \text{a. } & \frac{a^2 b^4 \times a^2 b^8}{a^3 b^2 \times a b^8} \\ &= \frac{a^4 b^{12}}{a^4 b^{10}} \\ &= a^0 b^2 \\ &= b^2 \\ &\longrightarrow \end{aligned}$$

$$\begin{aligned} \text{b. } & \frac{(c^5 g^2)(c^3 g)}{(gc)(gc^4)} \\ &= \frac{c^8 g^3}{g^2 c^5} \\ &= c^3 g \\ &\longrightarrow \end{aligned}$$

$$\begin{aligned} \text{c. } & \frac{8h^3 m^5 \times 3hm^2}{4m^2 h \times 6mh} \\ &= \frac{24h^4 m^7}{24h^2 m^3} \\ &= h^2 m^4 \\ &\longrightarrow \end{aligned}$$

$$\begin{aligned} \text{d. } & \frac{-2x^3 y \times 4x^2 y^6}{-4x^3 y \times -3xy^0} \\ &= \frac{-8x^5 y^7}{12x^4 y^1} \\ &= -\frac{2}{3}xy^6 \\ &\longrightarrow \end{aligned}$$

$$\begin{aligned} \text{e. } & \frac{(a^3 b^6)(ab^5)}{(ab)(a^3 b^4)} \\ &= \frac{a^4 b^9}{a^4 b^5} \\ &= b^4 \end{aligned}$$

$$\begin{aligned} \text{f. } & \frac{24g^2 h^4 \times g^7 h}{8h \times -3g^9 h^4} \\ &= \frac{24g^9 h^5}{-24g^9 h^5} \\ &= -1g^0 h^0 \\ &= -1 \\ &\longrightarrow \end{aligned}$$

# EXPONENTS LESSON 3

So far, we have the following rules:

$$a^b \times a^c = a^{b+c} \quad \text{and} \quad \frac{a^b}{a^c} = a^b \div a^c = a^{b-c}$$

We also know that  $a^0 = 1$ .

Today we are going to look at what happens when we have a higher power on the denominator.

Consider  $\frac{a^4}{a^7}$

Well, if, as we did last time, we write it out in full and divide the top and bottom repeatedly by  $a$  we get:

$$\frac{a \times a \times a \times a}{a \times a \times a \times a \times a \times a \times a} = \frac{\cancel{a^1} \times \cancel{a^1} \times \cancel{a^1} \times \cancel{a^1}}{\cancel{a^1} \times \cancel{a^1} \times \cancel{a^1} \times \cancel{a^1} \times a \times a \times a}$$

So, we end up with  $\frac{1}{a^3}$ .

However, our rule says that the answer should be  $a^{-3}$ .

In fact,  $a^{-3} = \frac{1}{a^3}$

This leads us to a new rule, actually a definition!

$$a^{-b} = \frac{1}{a^b}$$

This rule allows us to write numbers with positive exponents.

Let's consider a few examples:

$$m^{-5} = \frac{1}{m^5}$$

$$3^{-2} = \frac{1}{3^2}$$

what about  $\frac{5}{m^{-2}}$   
 $\rightarrow 5m^2$

So, we can see that when a **factor** moves from the top of the fraction to the bottom or vice versa then the **sign of its exponent changes**.

Let's write these expressions with positive exponents:

$$\begin{aligned} a^2 b^{-3} \\ = \frac{a^2}{b^3} \\ \longrightarrow \end{aligned}$$

$$\begin{aligned} \frac{g^5}{h^{-4}} \\ = \frac{g^5 h^4}{1} \\ \longrightarrow \end{aligned}$$

$$\begin{aligned} \frac{a^{-3} b^2}{2^{-1}} \\ = \frac{2 b^2}{a^3} \\ \longrightarrow \end{aligned}$$

It is very important to realise that we can only move a factor from the top to the bottom – NOT a term!

$$\frac{2a^{-1}}{b} = \frac{2}{ab} \quad \text{BUT} \quad \frac{2 + a^{-1}}{b} \neq \frac{2}{ab}$$

Note that we have choices when we do an example. Providing we obey our rules we can get the same, right answer in different ways. Consider the following example:

Simplify, giving your answers with positive exponents:

$$\frac{12a^3 b^{-4}}{6a^{-2} b^4}$$

$$\begin{aligned} \frac{12a^3 b^{-4}}{6a^{-2} b^4} \\ = \frac{12a^3 a^2}{6b^4 b^4} \\ = \frac{2a^5}{b^8} \\ \longrightarrow \end{aligned}$$

OR

$$\begin{aligned} \frac{12a^3 b^{-4}}{6a^{-2} b^4} \\ = 2a^5 b^{-8} \\ = \frac{2a^5}{b^8} \\ \longrightarrow \end{aligned}$$

## EXERCISE 1

Simplify, expressing your answers with positive exponents.

a.  $a^{-2}$   
 $= \frac{1}{a^2}$   
 $\longrightarrow$

b.  $3^{-1}$   
 $= \frac{1}{3}$   
 $\longrightarrow$

c.  $\frac{5}{g^{-4}}$   
 $= 5g^4$   
 $\longrightarrow$

d.  $-2a^{-6}b$   
 $= -\frac{2b}{a^6}$   
 $\longrightarrow$

e.  $3a^{-5}b^3c^{-4}$   
 $= \frac{3b^3}{a^5c^4}$   
 $\longrightarrow$

f.  $\frac{1}{5q^{-3}}$   
 $= \frac{q^3}{5}$   
 $\longrightarrow$

g.  $\frac{2^{-1}y^5g^{-3}}{3^{-2}}$   
 $= \frac{3^2y^5}{2g^3} \text{ or } \frac{9y^5}{2g^3}$   
 $\longrightarrow$

h.  $\frac{12b^{-3}}{4b^{-5}}$   
 $= 3b^2$   
 $\longrightarrow$

i.  $-4m^5 \times 3m^{-6}$   
 $= -12m^{-1}$   
 $= -\frac{12}{m} \text{ or } \frac{-12}{m}$   
 $\longrightarrow$

j.  $\frac{a^{-5}}{a^3}$   
 $= a^{-8}$   
 $= \frac{1}{a^8}$   
 $\longrightarrow$

k.  $-6x^{-8} \times 2x^5$   
 $= -12x^{-3}$   
 $= -\frac{12}{x^3}$   
 $\longrightarrow$

l.  $10p^{-5} \times 2p^7 \times 3p^{-4}$   
 $= 60p^{-2}$   
 $= \frac{60}{p^2}$   
 $\longrightarrow$

$$\begin{aligned}
 \text{m.} \quad & \frac{a^3 b^2 \times a b^{-4}}{a b^5 \times a^4 b^{-7}} \\
 &= \frac{a^4 b^{-2}}{a^5 b^{-2}} \\
 &= a^{-1} b^0 \\
 &= \frac{1}{a} \\
 &\longrightarrow
 \end{aligned}$$

$$\begin{aligned}
 \text{n.} \quad & \frac{-2x^3 y^6 \times 6xy}{-3x^6 y^{-3}} \\
 &= \frac{-12x^4 y^7}{-3x^6 y^{-3}} \\
 &= 4x^{-2} y^{10} \\
 &= \frac{4y^{10}}{x^2} \\
 &\longrightarrow
 \end{aligned}$$

$$\begin{aligned}
 \text{o.} \quad & \frac{(6x^3 y^5)(4x^2 y^9)}{(3x^4 y)(x^5 y^2)} \\
 &= \frac{24x^5 y^{14}}{3x^9 y^3} \\
 &= 8x^{-4} y^{11} \\
 &= \frac{8y^{11}}{x^4} \\
 &\longrightarrow
 \end{aligned}$$

$$\begin{aligned}
 \text{p.} \quad & \frac{a^{-3} b^7 c^5}{c^{-2} a^3 b^{-2}} \\
 &= \frac{b^2 b^7 c^5 c^2}{a^3 a^3} \\
 &= \frac{b^9 c^7}{a^6} \\
 &\longrightarrow
 \end{aligned}$$

$$\begin{aligned} \text{q.} \quad & \frac{5^{-1}x^0y^3}{y^{-2}} \\ & = \frac{y^5}{5} \\ & \longrightarrow \end{aligned}$$

$$\begin{aligned} \text{r.} \quad & \frac{2b^2c^5 \times 8b^6c}{4b^3c^6 \times 4b^0c^{-2}} \\ & = \frac{16b^8c^6}{16b^3c^4} \\ & = b^5c^2 \\ & \longrightarrow \end{aligned}$$

$$\begin{aligned} \text{s.} \quad & \frac{h^5 \times g^{-3}}{2^{-1}g^4h^7} \\ & = 2h^{-2}g^{-7} \\ & = \frac{2}{g^7h^2} \\ & \longrightarrow \end{aligned}$$

$$\begin{aligned} \text{t.} \quad & \frac{g^3h^{-2}}{h^3g^{-2}} \\ & = g^5h^{-5} \\ & = \frac{g^5}{h^5} \\ & \longrightarrow \end{aligned}$$

# EXPONENTS LESSON 4

So far, we have the following rules:

$$a^b \times a^c = a^{b+c} \quad \text{and} \quad \frac{a^b}{a^c} = a^b \div a^c = a^{b-c}$$

We also know that  $a^0 = 1$  and that  $a^{-b} = \frac{1}{a^b}$

We will now learn two more rules:

Consider $(ab)^2$	$(fgh)^3$
$(ab)^2$	$(fgh)^3$
$= (ab)(ab)$	$= (fgh)(fgh)(fgh)$
$= a^2 b^2$	$= f^3 g^3 h^3$
$\longrightarrow$	$\longrightarrow$

We can summarise these in words as follows:

When a product (a multiplication sum) is raised to a power then **all** the factors get the power!

We can write this in symbols as follows:

$$(ab)^c = a^c b^c$$

Note that  $(a+b)^2 \neq a^2 + b^2$

Now consider what happens when a quotient (a division sum) is raised to a power:

$\left(\frac{g}{h}\right)^2$	$\left(\frac{a}{b}\right)^3$
$= \left(\frac{g}{h}\right)\left(\frac{g}{h}\right)$	$= \left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)$
$= \frac{g^2}{h^2}$	$= \frac{a^3}{b^3}$
$\longrightarrow$	$\longrightarrow$

We can summarise these in words as follows:

When a quotient (a multiplication sum) is raised to a power then **both top (numerator) and the bottom (denominator)** get the power!

We can write this in symbols as follows:

$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$



## EXERCISE 1

Simplify, expressing answers with positive exponents where necessary:

$$\begin{aligned}\text{a. } & \frac{(xy)^3 \times 9x^2y}{x^3y^6 \times 3} \\ &= \frac{x^3y^3 \times 9x^2y}{3x^3y^6} \\ &= \frac{9x^5y^4}{3x^3y^6} \\ &= 3x^2y^{-2} \\ &= \frac{3x^2}{y^2}\end{aligned}$$

$$\begin{aligned}\text{b. } & \frac{(2xy)^2 \times 3y^3x^5}{24x^3y^6} \\ &= \frac{4x^2y^2 \times 3y^3x^5}{24x^3y^6} \\ &= \frac{12x^7y^5}{24x^3y^6} \\ &= \frac{x^4y^{-1}}{2} \\ &= \frac{x^4}{2y} \\ &\longrightarrow\end{aligned}$$

$$\begin{aligned}\text{c. } & (2ab)^3 \div 4a^2b \\ &= \frac{8a^3b^3}{4a^2b} \\ &= 2ab^2 \\ &\longrightarrow\end{aligned}$$

$$\begin{aligned}\text{d. } & \frac{2t^4y^8}{(2yt)^3} \\ &= \frac{2t^4y^8}{8y^3t^3} \\ &= \frac{ty^5}{4} \\ &\longrightarrow\end{aligned}$$

$$\begin{aligned}\text{e. } & \left(\frac{a}{b}\right)^3 \times 3a^3b^6 \\ &= \frac{a^3}{b^3} \times 3a^3b^6 \\ &= 3a^6b^3 \\ &\longrightarrow\end{aligned}$$

$$\begin{aligned}\text{f. } & \frac{x^4y^3 \times 2x^{-5}y^8}{\left(\frac{y}{x}\right)^6 \times x^3y} \\ &= \frac{2x^{-1}y^{11}}{\frac{y^6}{x^6} \times x^3y} \\ &= \frac{2x^{-1}y^{11}}{y^7x^{-3}} \\ &= 2x^2y^4 \\ &\longrightarrow\end{aligned}$$

Now let's look at what happens when a number which already has a power is raised to a power:

Consider  $(x^2)^3$

Well, we know that cubing a quantity involves multiplying 3 lots of itself by one another:

$$\text{So, } (x^2)^3 = x^2 \times x^2 \times x^2 = x^6$$

Try doing the following in a similar way. See if you can make up a rule!

$$\begin{aligned} (a^4)^2 \\ = a^4 a^4 \\ = a^8 \\ \longrightarrow \end{aligned}$$

$$\begin{aligned} (t^2)^5 \\ = t^2 t^2 t^2 t^2 t^2 \\ = t^{10} \\ \longrightarrow \end{aligned}$$

$$\begin{aligned} (g^5)^4 \\ = g^5 g^5 g^5 g^5 \\ = g^{20} \\ \longrightarrow \end{aligned}$$

Indeed, when we have a number with a power being raised to another power, we

multiply the powers.

We can write this with symbols as follows:

$$(a^b)^c = a^{bc}$$

That brings us to the end of all the rules we need for exponents in Grade 9.

Let's right them all down:

Exponents rule! I mean exponent rules!

$$a^b \times a^c = a^{b+c} \quad \text{and} \quad \frac{a^b}{a^c} = a^b \div a^c = a^{b-c}$$

$$(ab)^c = a^c b^c \quad \text{but note that } (a+b)^2 \neq a^2 + b^2$$

$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

$$(a^b)^c = a^{bc}$$

$$\text{We also know that } a^0 = 1 \text{ and that } a^{-b} = \frac{1}{a^b}$$

## EXERCISE 2

Simplify expressing your answers with positive exponents where necessary:

$$\begin{aligned}\text{a. } & (a^2b)^3 \times a^{-4}b^{-7} \\ & = a^6b^3 \times a^{-4}b^{-7} \\ & = a^2b^{-4} \\ & = \frac{a^2}{b^4} \\ & \longrightarrow\end{aligned}$$

$$\begin{aligned}\text{b. } & \left(\frac{y^3}{x^2}\right)^{-2} \\ & = \frac{y^{-6}}{x^{-4}} \\ & = \frac{x^4}{y^6} \\ & \longrightarrow\end{aligned}$$

$$\begin{aligned}\text{c. } & \frac{(4x^3)^2}{4x^{-2}} \\ & = \frac{16x^6}{4x^{-2}} \\ & = 4x^8 \\ & \longrightarrow\end{aligned}$$

$$\begin{aligned}\text{d. } & \frac{(x^2y)^3 \times 2xy}{x^{-2}y^6 \times 4x^4y} \\ & = \frac{x^4y^6 \times 2xy}{4x^2y^7} \\ & = \frac{2x^5y^7}{4x^2y^7} \\ & = \frac{x^3}{2} \\ & \longrightarrow\end{aligned}$$

$$\begin{aligned}\text{e. } & \frac{(3x^2y)^2 \times xy^0}{2x^2y} \\ & = \frac{9x^4y^2 \times x}{2x^2y} \\ & = \frac{9x^5y^2}{2x^2y} \\ & = \frac{9x^3y}{2} \\ & \longrightarrow\end{aligned}$$

$$\begin{aligned}\text{f. } & \frac{(7xy)^5 \times x^3y}{(7x^2y)^4 \times x^0} \\ & = \frac{7^5x^5y^5 \times x^3y}{7^4x^8y^4} \\ & = \frac{7^5x^8y^6}{7^4x^8y^4} \\ & = 7y^2 \\ & \longrightarrow\end{aligned}$$

$$\begin{aligned}\text{g. } & \left(\frac{(x^3yh)^3 \times (xhy)^4}{x^3yh \times x^7hy^5}\right)^0 \\ & = 1 \\ & \longrightarrow\end{aligned}$$

$$\begin{aligned}\text{h. } & 2 + 2^{-1} + 2^0 \\ & = 2 + \frac{1}{2} + 1 \\ & = 3\frac{1}{2} \\ & \longrightarrow\end{aligned}$$

$$\begin{aligned}\text{i. } & 11x^0 \div (11x)^0 \\ & = \frac{11}{1} \\ & = 11 \\ & \longrightarrow\end{aligned}$$

$$\begin{aligned}\text{j. } & \left(\frac{x^2y^{-2}}{xy^{-5}}\right)^{-1} \\ & = \frac{x^{-2}y^2}{x^{-1}y^5} \\ & = x^{-1}y^{-3} \\ & = \frac{1}{xy^3} \\ & \longrightarrow\end{aligned}$$

### EXERCISE 3

You are helping a friend with exponents. He has attempted the following questions but has made at least one error in each of them. Identify the error and give the correct answer:

a. 
$$\frac{1}{-3x^{-3}}$$
$$= 3x^3$$

$$\frac{1}{-3} \neq 3!$$

correct answer

$$\frac{x^3}{-3}$$
  
→

b. 
$$2^5 \times 2^{-1}$$
$$= 4^4$$

don't multiply bases!

correct answer:

$$2^4$$
  
→

c. 
$$\frac{2x^3y^{-1}}{6x^2y^{-2}}$$
$$= \frac{x}{4y}$$

$$\frac{2}{6} \neq \frac{1}{4}$$

also 
$$\frac{y^{-1}}{y^{-2}} \neq \frac{1}{y}$$

correct answer:

$$\frac{x}{3y}$$
  
→

d. 
$$(3x^2)^3$$
$$= 9x^5$$

when applying a power to an existing power we multiply them we don't add them!

correct answer:

$$9x^6$$
  
→

e. 
$$\frac{-10m^4}{-5m^0}$$
$$= -2m^3$$

$$-\frac{10}{-5} \neq -2 \text{ and } \frac{m^4}{m^0} \neq m^3$$

correct answer:

$$2m^4$$
  
→

f. 
$$a^2b^0$$
$$= 1$$

the power 0 is only on the  $b$

correct answer:

$$a^2$$
  
→