Resource for Day 1 – Algebra and Equations – Thursday 19 October

1. Simplify the following:

a. $\dfrac{(2a^2b)^3 \times 3a^3b^{-3}}{4a^3b \times 6a^2b^8}$ (4)

$= \dfrac{8a^6b^3 \times 3a^3b^{-3}}{24a^5b^9}$

$= \dfrac{24a^9}{24a^5b^9}$

$= \dfrac{a^4}{b^9}$ $\longrightarrow$

b. $\dfrac{12^{x+1} \times 3^{x-1}}{36^{x-1}}$ (4)

$= \dfrac{(2^2.3)^{x+1} \times 3^{x-1}}{(2^2.3^2)^{x-1}}$

$= \dfrac{2^{2x+2}.3^{x+1} \times 3^{x-1}}{2^{2x-2}.3^{2x-2}}$

$= \dfrac{2^{2x+2}.3^{2x}}{2^{2x-2}.3^{2x-2}}$

$= 2^4 \times 3^2$

$= 144$ $\longrightarrow$

c. $\dfrac{4^{x+1} - 2^{2x}}{2^{2x+2}}$ (3)

$= \dfrac{(2^2)^{x+1} - 2^{2x}}{2^{2x+2}}$

$= \dfrac{2^{2x+2} - 2^{2x}}{2^{2x+2}}$

$= \dfrac{2^{2x}(2^2 - 1)}{2^{2x+2}}$

$= \dfrac{3}{2^2}$

$= \dfrac{3}{4}$ $\longrightarrow$

d. $\dfrac{\sqrt{12} - \sqrt{8}}{\sqrt{27} - \sqrt{18}}$ (3)

$= \dfrac{\sqrt{4 \times 3} - \sqrt{4 \times 2}}{\sqrt{9 \times 3} - \sqrt{9 \times 2}}$

$= \dfrac{2\sqrt{3} - 2\sqrt{2}}{3\sqrt{3} - 3\sqrt{2}}$

$= \dfrac{2\left(\sqrt{3} - \sqrt{2}\right)}{3\left(\sqrt{3} - \sqrt{2}\right)}$

$= \dfrac{2}{3}$ $\longrightarrow$

e. $\dfrac{4x^3 + 6x^2 - 8x - 2}{2x - 2}$ (4)

$= \dfrac{2(2x^3 + 3x^2 - 4x - 1)}{2(x - 1)}$

$= \dfrac{2(x - 1)(2x^2 + 5x + 1)}{2(x - 1)}$

$= \dfrac{\cancel{2^1(x-1)^1}(2x^2 + 5x + 1)}{\cancel{2^1(x-1)^1}}$

$= 2x^2 + 5x + 1$ $\longrightarrow$

2. Solve the following:

a. $2x^{\frac{3}{5}} - 14 = 30$ (3)

$2x^{\frac{3}{5}} = 44$

$x^{\frac{3}{5}} = 22$

$x = 22^{\frac{5}{3}}$

$x \approx 172{,}73$

b. $\frac{x}{a} + \frac{bx}{c} = d$ (3)

$xc + bax = dac$

$x(c + ba) = dac$

$x = \frac{dac}{c + ba}$

c. $x^3 - 16x = 0$ (3)

$x(x^2 - 16) = 0$

$x(x + 4)(x - 4) = 0$

$x = 0$ or $x = 4$ or $x = -4$

d. $(x - 3)(x + 2) = 6$ (3)

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

$x = 4$ or -3

e. $x^2 - 4x - 9 = 0$ (3)

by completing the square

$x^2 - 4x - 9 = 0$

$x^2 - 4x = 9$

$(x - 2)^2 = 9 + 4$

$(x - 2)^2 = 13$

$x - 2 = \pm\sqrt{13}$

$x = 2 \pm \sqrt{13}$

f. $2x^2 + 5x - 9 = 0$ (5)

by completing the square

$2x^2 + 5x - 9 = 0$

$x^2 + \frac{5}{2}x = \frac{9}{2}$

$\left(x + \frac{5}{4}\right)^2 = \frac{9}{2} + \frac{25}{16}$

$\left(x + \frac{5}{4}\right)^2 = \frac{97}{16}$

$x + \frac{5}{4} = \pm\frac{\sqrt{97}}{4}$

$x = \frac{-5 \pm \sqrt{97}}{4}$

g. $x^2 + px - \frac{3p^2}{4} = 0$ (3)

$x = \frac{-p \pm \sqrt{p^2 - 4(1)\left(-\frac{3}{4}\right)}}{2(1)}$

$x = \frac{-p \pm \sqrt{4p^2}}{2}$

$x = -\frac{3p}{2}$ or $\frac{p}{2}$

h. $\sqrt{2x - 1} + x = 8$ (5)

$\sqrt{2x - 1} = 8 - x$

$2x - 1 = (8 - x)^2$

$2x - 1 = 64 - 16x + x^2$

$x^2 - 18x + 65 = 0$

$(x - 13)(x - 5) = 0$

$x = 13$ or 5

but a check reveals $x = 5$ only

i. (4)

$9^{x+2} = 27^{2x+1}$

$(3^2)^{x+2} = (3^3)^{2x+1}$

$3^{2x+4} = 3^{6x+3}$

$2x + 4 = 6x + 3$

$1 = 4x$

$x = \frac{1}{4}$ ⟶

j. (2)

$5^{x+1} = 20$

$x + 1 = \log_5 20$

$x = \log_5 20 - 1$

$x = 0{,}86$ ⟶

k. (4)

$7^{x+1} - 2(7^x) = 85$

$7^x(7 - 2) = 85$

$7^x = 17$

$x = \log_7 17$

$x = 1{,}46$ ⟶

l. (6)

$3^x - 4 - 7(3^{1-x}) = 0$

$3^x - 4 - 7\left(\frac{3}{3^x}\right) = 0$

$3^{2x} - 4.3^x - 21 = 0$

$(3^x - 7)(3^x + 3) = 0$

$3^x = 7$ or $3^x = -3$ (impossible)

$x = \log_3 7$

$x = 1{,}77$ ⟶

m. (3)

$\log_2(x^2 + 2x) = 3$

$x^2 + 2x = 8$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = -4$ or $x = 2$ ⟶

n. (3)

$\log_{x+2} 16 = 2$

$(x + 2)^2 = 16$

$x + 2 = \pm 4$

$x = 2$ or -6

$x = -6$ undefined

$x = 2$ only ⟶

o. (5)

$x(3 - x) > -10$

$3x - x^2 + 10 > 0$

$x^2 - 3x - 10 < 0$

$(x - 5)(x + 2) < 10$

$-2 < x < 5$ ⟶

p. (5)

$3x + 5y = 19$ ①

$4x - 2y = 8$ ②

4 × ① gives $12x + 20y = 76$ ③

3 × ② gives $12x - 6y = 24$ ④

③ − ④ gives:

$26y = 52$

$y = 2$

$x = 3$

(3;2) ⟶

3. Find k if the equation $x^2 + kx + \frac{k}{2} + 2 = 0$ has non-real roots. (4)

$x^2 + kx + \frac{k}{2} + 2 = 0$

if no real roots then $\Delta < 0$

$\Delta = k^2 - 4(1)\left(\frac{k}{2} + 2\right) = k^2 - 2k - 8 < 0$

$(k-4)(k+2) < 0$

$-2 < k < 4$