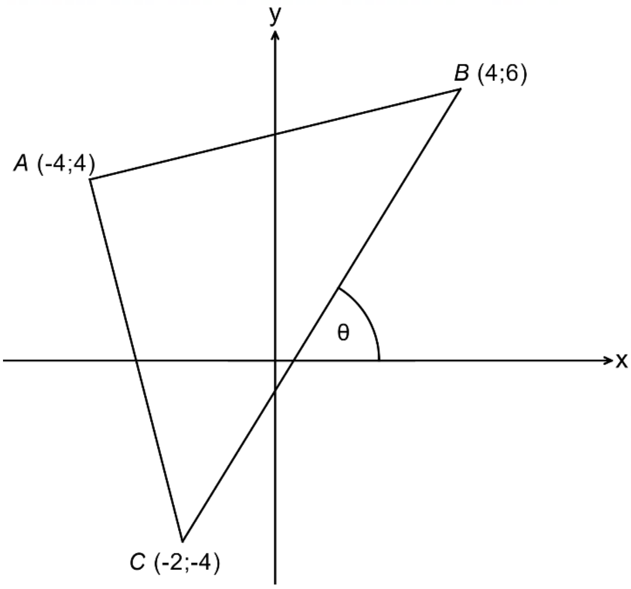
SHAYA IZIBALO MATHS INSTITUTE in partnership with KWAZULU-NATAL PROVINCE EDUCATION REPUBLIC OF SOUTH AFRICA Supported by

Resource for Day 2 – Analytical Geometry – Friday 20 October

1. Consider the diagram and answer the questions alongside:



a. Prove that $B\hat{A}C = 90°$ (3)

$mAB = \frac{1}{4}$ and $mAC = -4$

since $mAB \times mAC = -1$

$AB \perp AC$ or $B\hat{A}C = 90°$

b. Determine area ΔABC (3)

Area $\Delta ABC = \frac{1}{2} base \times ht$

Area $\Delta ABC = \frac{1}{2} AC \times AB$

Area $\Delta ABC = \frac{1}{2}\sqrt{8^2 + 2^2} \times \sqrt{8^2 + 2^2}$

Area $\Delta ABC = 34\ u^2$

c. Find the midpoint of BC (2)

(1;1)

d. Find the size of θ (2)

$$mCB = \frac{10}{6} = \frac{5}{3}$$

$$\tan\theta = \frac{5}{3}$$

$$\theta = \tan^{-1}\left(\frac{5}{3}\right)$$

$$\theta = 59°$$

e. Find the equation of *AB* (3)

$$y - 6 = \frac{1}{4}(x - 4)$$

$$y = \frac{1}{4}x + 5$$

f. Find the coordinates of *D* if *ABDC* is a parallelogram (3)

$D(6;-2)$

g. Find the equation of a circle centre A which has the *y*-axis as a **tangent** (3)

$$(x+4)^2 + (y-4)^2 = 16$$

h. Find point E if $AB : AE = 1 : 3$ (4)

B is $\frac{1}{3}$ of the way along AE

change in x from A to B is 8 so change in x from B to E is 16
change in y from A to B is 2 so change in y from B to E is 4
$E(20;10)$

i. Find value(s) of *k* if $F(k;9)$ is 5 units from B. (4)

$$(k-4)^2 + (9-6)^2 = 25$$

$$(k-4)^2 + 9 = 25$$

$$(k-4)^2 = 16$$

$$k - 4 = \pm 4$$

$$k = 4 \pm 4$$

$$k = 0 \text{ or } 8$$

2. Find the equation of the tangent to the circle $x^2 + y^2 - 6x + 4y = 19$ at the point $A(7;2)$ (5)

$x^2 + y^2 - 6x + 4y = 19$

$(x-3)^2 + (y+2)^2 = 19 + 9 + 4$

centre (3;-2)

$m_{radius} = 1$

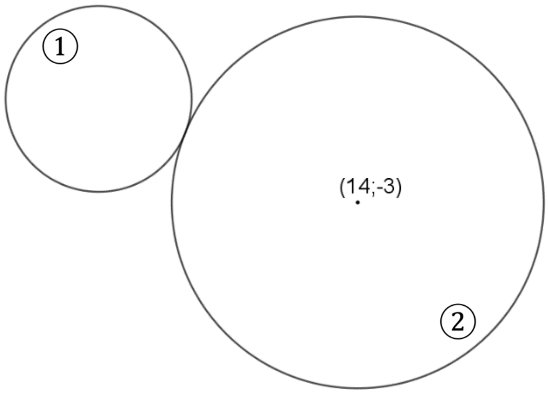
$m_{tangent} = -1$ (rad ⊥ tangent)

$y - 2 = -1(x - 7)$

$y = -x + 9$

3. Consider the circle $x^2 + 2x + y^2 - 6y = 19$. It is depicted below as circle ①

Find the equation of circle ② which has a centre at (14;-3) and **touches** circle ① (6)



$x^2 + 2x + y^2 - 6y = 19$

$(x+1)^2 + (y-3)^2 = 19 + 1 + 9$

so, center is (-1;3) and $r = \sqrt{29}$

distances between centres is $\sqrt{15^2 + 6^2} = \sqrt{261}$

since they **touch** the distances between centres must be sum of radii

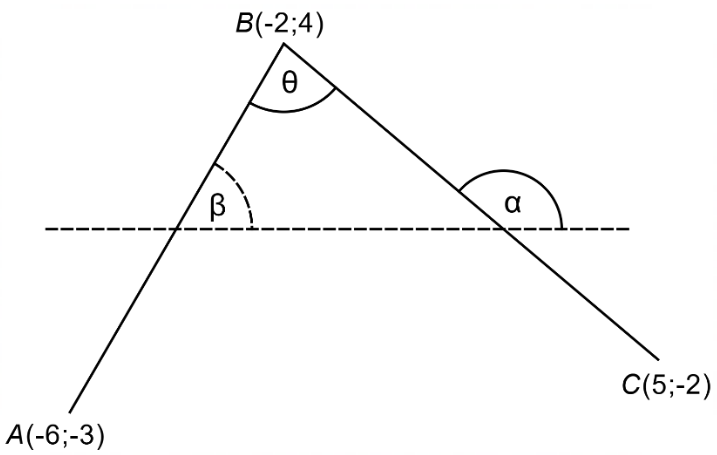
so, radius of circle 2 is $r = \sqrt{261} - \sqrt{29}$

$r^2 = \left(\sqrt{261} - \sqrt{29}\right)^2$

$r^2 = 116$

$\therefore (x-14)^2 + (y+3)^2 = 116$

4. Find θ in the diagram below: (6)



$mAB = \frac{7}{4}$ and $mBC = -\frac{6}{7}$

$\therefore \tan\beta = \frac{7}{4}$ and $\tan\alpha = -\frac{6}{7}$

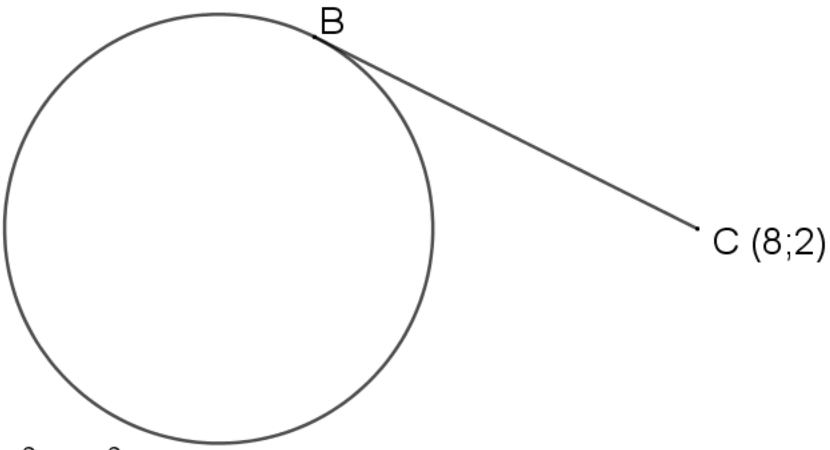
$\therefore \beta = \tan^{-1}\left(\frac{7}{4}\right)$ and $\alpha = 180° - \tan^{-1}\left(\frac{6}{7}\right)$

$\therefore \beta = 60{,}26°$ and $\alpha = 139{,}4°$

now $\theta = \alpha - \beta$ (ext. ∠ of Δ)

$\therefore \theta = 79{,}14°$

5. Find the length of the tangent below



$x^2 + y^2 - 6x - 4y + 8 = 0$

$(x-3)^2 + (y-2)^2 = -8 + 9 + 4$

centre is $O(3;2)$ and $r^2 = 5$

now $OC^2 = 25$

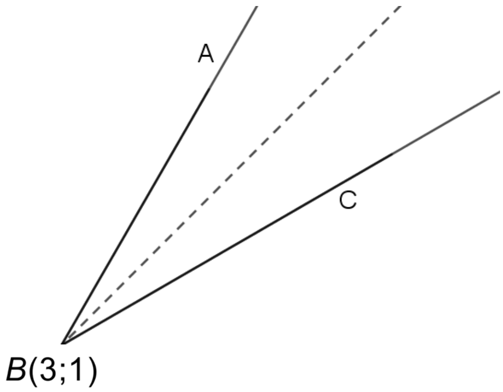
and $BC^2 + r^2 = OC^2$ (Pythagoras, rad ⊥ tangent)

$BC^2 + 5 = 25$

$BC^2 = 20$

$BC = \sqrt{20} \approx 4{,}47$ units

6. The dashed line is the bisector of $A\hat{B}C$. Find its equation if the gradient $AB = \sqrt{3}$ and the gradient of $BC = \dfrac{\sqrt{3}}{3}$



$\angle$ of inclination of $AB = \tan^{-1}\sqrt{3} = 60°$

$\angle$ of inclination of $BC = \tan^{-1}\dfrac{\sqrt{3}}{3} = 30°$

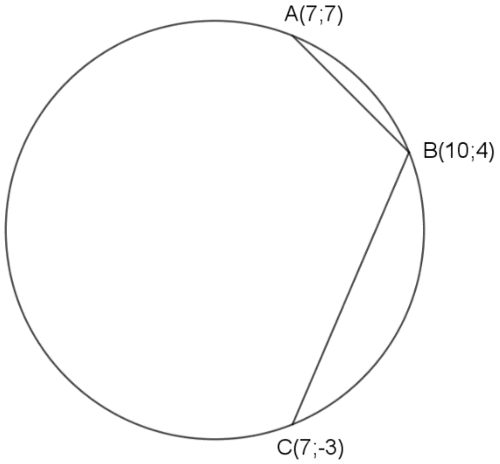
$\therefore A\hat{B}C = 30°$ and $\angle$ of inclination of angle bisector is $45°$

so, gradient of angle bisector is $\tan 45° = 1$

$y - 1 = 1(x-3)$

$y = x - 2$

7. Find the centre of the circle below. Hint: the perpendicular bisector of a chord passes through the centre of the circle!



midpoint of $AB = (8{,}5;5{,}5)$ and gradient of $AB = -1$

so $\perp$ bisector of AB has a gradient of 1

and equation of $\perp$ bisector of AB is

$y - 5{,}5 = 1(x - 8{,}5)$ or $y = x - 3$ ①

midpoint of BC is $(8{,}5\ ;\ 0{,}5)$ and gradient of $BC = \frac{7}{3}$

so $\perp$ bisector of BC has a gradient of $-\frac{3}{7}$

and equation of $\perp$ bisector of BC is

$y - 0{,}5 = -\frac{3}{7}(x - 8{,}5)$ or $y = -\frac{3}{7}x + \frac{29}{7}$ ②

solving ① and ② simultaneously gives:

$x - 3 = -\frac{3}{7}x + \frac{29}{7}$

$7x - 21 = -3x + 29$

$10x = 50$

$x = 5$ and $y = 2$

centre (5;2)