SHAYA IZIBALO MATHS INSTITUTE in partnership with KWAZULU-NATAL PROVINCE EDUCATION REPUBLIC OF SOUTH AFRICA Supported by MICHAELHOUSE COMMUNITY PARTNERSHIPS

Resource for Day 3 – Finance and Growth – Monday 23 October

1. Water weed in a dam increases by 6% per month. If the initial area covered by weed is 20 m^2 then what is the area covered after 8 months? (3)

   our multiplier is 1,06 or 106%, so
   after 1 month: $20 \times 1{,}06$
   after 2 months: $20 \times 1{,}06^2$
   after 8 months: $20 \times 1{,}06^8 = 31{,}88\ m^2$

2. A share portfolio loses 3% of its value per year. If it is initially worth R27 000 what will it be worth after 10 years? (3)

   our multiplier is 0,97 or 97%, so
   after 1 year: $27\ 000 \times 0{,}97$
   after 2 years: $27\ 000 \times 0{,}97^2$
   after 10 years: $27\ 000 \times 0{,}97^{10} = R19\ 910{,}45$

3. An item costs R397,44 with 15% VAT included. What did it cost before the VAT was added? (3)

   suppose initial price is P. Our multiplier to add 15% VAT is 1,15 or 115%
   so, $1{,}15P = 397{,}44$
   $\therefore P = \dfrac{397{,}44}{1{,}15}$
   $\therefore P = R345{,}60$

4. A shopkeeper marks up an item by 20% but then offers a 20% discount. What is her net profit or loss expressed as a percentage? (3)

   suppose initial price is P.
   Our multiplier to add 20% is 1,2 or 120%
   Our multiplier to discount by 20% is 0,8 or 80%
   now $P \times 1{,}2 \times 0{,}8 = 0{,}96P$
   A resulting multiplier of 0,96 or 96% corresponds to 4% loss!

5. An item depreciates at 8% per annum according to the reducing balance method. After how many years (to the nearest year) will it be worth 25% of its original value? (3)

$$A = P(1-i)^n$$

$$\frac{p}{4} = p(1-0{,}08)^n$$

$$\frac{1}{4} = 0{,}92^n$$

$$n = \log_{0{,}92}\left(\frac{1}{4}\right)$$

$$\underline{n = 17 \text{ years}}$$

6. What annual rate compounded quarterly will give the same return as 10% p.a. compounded semi-annually (every 6 months)? (4)

$R1$ for 1 year!

$$1\left(1+\frac{i}{4}\right)^4 = 1\left(1+\frac{0{,}1}{2}\right)^2$$

$$\left(1+\frac{i}{4}\right)^4 = (1{,}05)^2$$

$$1+\frac{i}{4} = \left((1{,}05)^2\right)^{\frac{1}{4}}$$

$$1+\frac{i}{4} = (1{,}05)^{\frac{1}{2}}$$

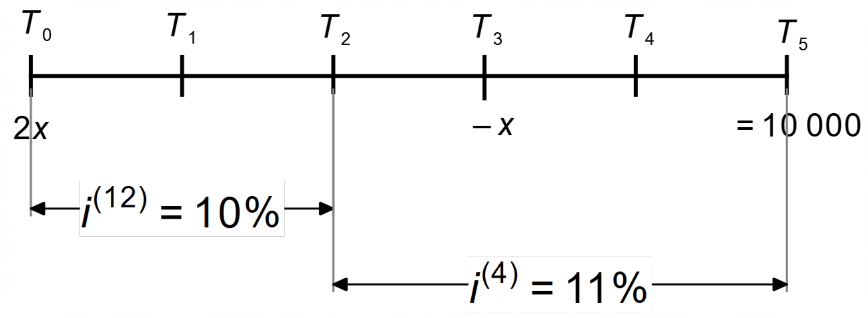
$$\frac{i}{4} = (1{,}05)^{\frac{1}{2}} - 1$$

$$i = 4\left((1{,}05)^{\frac{1}{2}} - 1\right)$$

$$i = 0{,}09878031$$

$$\underline{i = 9{,}878\%}$$

7. Thabo deposits $2x$ into an account. Three years later he withdraws x. Five years after his initial deposit he has R10 000. His money earned interest of 10% p.a. compounded monthly for the first 2 years and then 11% p.a. compounded quarterly for the remainder of the time. Find x showing all working details. (7)



Either – work along the time line to the end

At T_2 : $2x\left(1+\frac{0,1}{12}\right)^{24}$

At T_3 : $\left(2x\left(1+\frac{0,1}{12}\right)^{24}\right)\left(1+\frac{0,11}{4}\right)^{4}-x$

At T_5 : $\left(\left(2x\left(1+\frac{0,1}{12}\right)^{24}\right)\left(1+\frac{0,11}{4}\right)^{4}-x\right)=10\ 000$

This is ugly to solve. This method works well if we have numbers rather than variables.

OR – consider the different withdrawals and deposits separately!

$$2x\left(1+\frac{0,1}{12}\right)^{24}\left(1+\frac{0,11}{4}\right)^{12}-x\left(1+\frac{0,11}{4}\right)^{8}=10\ 000$$

$$x\left[2\left(1+\frac{0,1}{12}\right)^{24}\left(1+\frac{0,11}{4}\right)^{12}-\left(1+\frac{0,11}{4}\right)^{8}\right]=10\ 000$$

$$x=\frac{10\ 000}{2\left(1+\frac{0,1}{12}\right)^{24}\left(1+\frac{0,11}{4}\right)^{12}-\left(1+\frac{0,11}{4}\right)^{8}}$$

$x = R4\ 678,20$

8. Nonhlanhla deposits R3000 per month for 5 years months into an account earning 9% p.a. compounded monthly. How much is in the account 3 months after her final payment? (5)

$$F_v = x\left[\frac{(1+i)^n - 1}{i}\right]$$

$$F_v = 3\,000\left[\frac{\left(1+\frac{0{,}09}{12}\right)^{60} - 1}{\frac{0{,}09}{12}}\right]$$

$$F_v = R226\,272{,}41$$

but this is the balance immediately after making her last payment
so we must now add 3 months of compound interest to get:

$$F_v = 226\,272{,}41\left(1+\frac{0{,}09}{12}\right)^{3}$$

$$\underline{F_v = 231\,401{,}82}$$

9. Sanele borrows R100 000. He repays it with monthly repayments over 5 years starting one month from the date of the loan. Interest is charged on the outstanding balance at 12% p.a. compounded monthly.

- How much is his monthly repayment? (5)

$$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$100\,000 = x\left[\frac{1-\left(1+\frac{0{,}12}{12}\right)^{-60}}{\frac{0{,}12}{12}}\right]$$

$$x = 100\,000 \div \left[\frac{1-\left(1+\frac{0{,}12}{12}\right)^{-60}}{\frac{0{,}12}{12}}\right]$$

$$\underline{x = R2\,224{,}44}$$

- How much does he still owe after making his 36th payment? (5)

$OB =$ loan with interest $-$ F_v of payments made with interest

$$= 100\,000\left(1+\frac{0{,}12}{12}\right)^{36} - 2\,224{,}44\left[\frac{\left(1+\frac{0{,}12}{12}\right)^{36} - 1}{\frac{0{,}12}{12}}\right]$$

$$= 143\,076{,}88 - 95\,821{,}93$$

$$= \underline{R47\,254{,}95}$$

- How is his 37th payment split between interest and capital? (2)

the 37th payment services (pays) the **INTEREST** charged on the outstanding balance after 36 payments.

This is $47\ 254{,}95\left(\frac{0{,}12}{12}\right) = R472{,}55$

We can work this out as a percentage of the payment of $R2\ 224{,}44$

$\frac{472{,}55}{2\ 224{,}44} \times 100 = 21{,}24\%$

so, $21{,}24\%$ of the 37th payment goes to **INTEREST** and $78{,}76\%$ to **CAPITAL**

- How are his first three years of payments split between interest and capital? (2)

The outstanding balance has dropped from $R100\ 000$ to $R47\ 254{,}95$

This amount is $R52\ 745{,}05$ and is **CAPITAL**

It is out of 36 payments of $R2\ 224{,}44$ – a total of $80\ 079{,}84$

We can work it out as a percentage....

$\frac{52\ 745{,}05}{80\ 079{,}84} \times 100 = 65{,}87\%$

So, of the first three years' payments:

$65{,}87\%$ went to **CAPITAL** and $34{,}13\%$ went to **INTEREST**

10. Mrs Tholanah borrows R50 000. Interest of 10% p.a. compounded monthly is charged on the outstanding balance. She makes monthly repayments of R1000. How many payments will it take to settle the loan? (6)

$$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$50\ 000 = 1\ 000\left[\frac{1-\left(1+\frac{0,1}{12}\right)^{-n}}{\frac{0,1}{12}}\right]$$

$$50 = \frac{1-\left(1+\frac{0,1}{12}\right)^{-n}}{\frac{0,1}{12}}$$

$$50 \times \frac{0,1}{12} = 1-\left(1+\frac{0,1}{12}\right)^{-n}$$

$$\left(1+\frac{0,1}{12}\right)^{-n} = 1-50\times\frac{0,1}{12}$$

$$-n = \log_{1+\frac{0,1}{12}}\left(1-50\times\frac{0,1}{12}\right)$$

$$n = -\log_{1+\frac{0,1}{12}}\left(1-50\times\frac{0,1}{12}\right)$$

$n = 64,95$

so, 65 payments are required

11. Nathi has a construction business for which he buys a TLB costing R1 250 000. He plans to use a sinking fund to make provision for the replacement of the TLB in 5 years' time. Over this period the following considerations apply:

- ✓ The price of a new TLB is expected to increase by 6% p.a. due to inflation.
- ✓ His existing TLB will depreciate at 7,5% p.a. using the reducing balance method. He will trade it in at its depreciated value after 5 years.
- ✓ His sinking fund will earn 9% p.a. compounded monthly.

How much must Nathi invest per month in order to be able to buy the new TLB when the time comes? (8)

The new TLB will cost $1\,250\,000(1+0{,}06)^5 = R1\,672\,781{,}97$

The old TLB will be worth $1\,250\,000(1-0{,}075)^5 = R846\,483{,}85$

So, the shortfall to be met by the sinking fund is

$1\,672\,781{,}97 - 846\,483{,}85 = R826\,298{,}12$

$$F_v = x\left[\frac{(1+i)^n - 1}{i}\right]$$

$$826\,298{,}12 = x\left[\frac{\left(1+\frac{0{,}09}{12}\right)^{60} - 1}{\frac{0{,}09}{12}}\right]$$

$$x = 826\,298{,}12 \div \left[\frac{\left(1+\frac{0{,}09}{12}\right)^{60} - 1}{\frac{0{,}09}{12}}\right]$$

$x = 10\,955{,}35$

Nathi must invest $R10\,955{,}35$ per month