Resource for Day 4 - Calculus – Tuesday 24 October

1. Given $f(x) = (2x+3)^2$, determine $f'(x)$ by first principles. (5)

2. Given $f(x) = \frac{1}{x}$, determine $f'(x)$ by first principles. (6)

3. Determine the following derivatives, giving your answers with positive exponents where necessary. Remember to pay careful attention to notation!

   a. $y = \frac{x^3 + 5x^2 + 15x + 18}{2x + 4}$ (4)

   b. $D_x\left[\sqrt[5]{x^7} + \frac{4}{x^3} + \sqrt{2}\right]$ (4)

   c. $\frac{d}{dx}\left(\frac{4x^9 + 5x}{2x^7}\right)$ (3)

4. Determine whether $f(x) = x - \sqrt{x}$ is increasing or decreasing or stationary when $x = 2$? Justify your answer. (3)

5. Determine for which value(s) of x the function $y = 2x^3 + 5x^2 - 4x - 3$ is increasing? (5)

6. Determine for which value(s) of x the function $f(x) = \frac{x^4}{12} + \frac{x^3}{6} - x^2 + 8x + 4$ is concave down. (5)

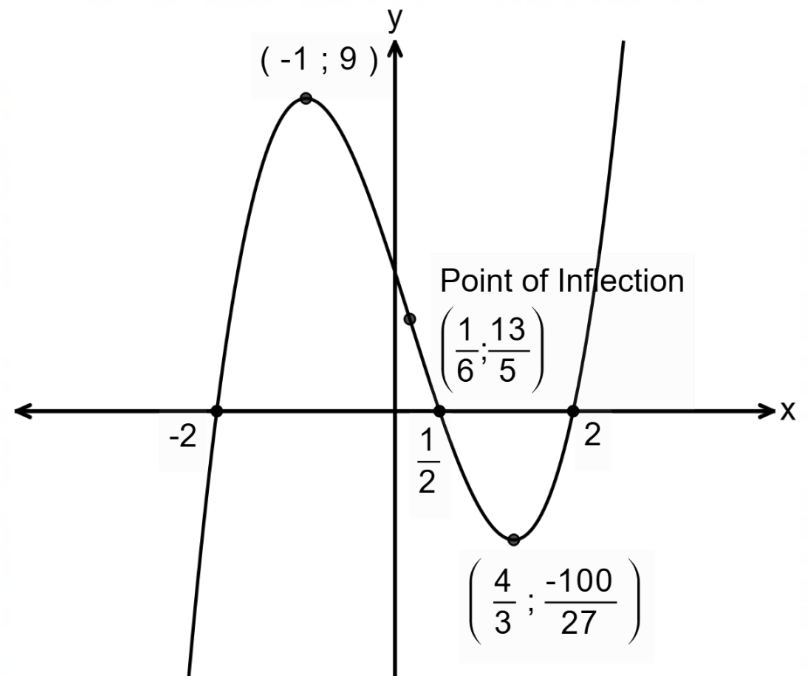
7. Find the equation of the tangent to the function $y = \frac{6}{x} - 2$ at its x-intercept. (4)

8. The function $f(x) = x^3 - x^2 - x - 1$ has two tangents which are parallel to the line $2y - 8x = 16$. Find both of their equations. (6)

9. The function $f(x) = x^3 + bx^2 + cx + 5$ has a local minimum at $(2;-3)$. Find b and c. (6)

10. Determine the **average gradient** of the function $y = 2^x - 8$ between its intercepts with the axes. (4)

11. Consider the graph of the function $f(x)$ shown below and use it to solve the equations and inequalities which follow:



Solve for x:

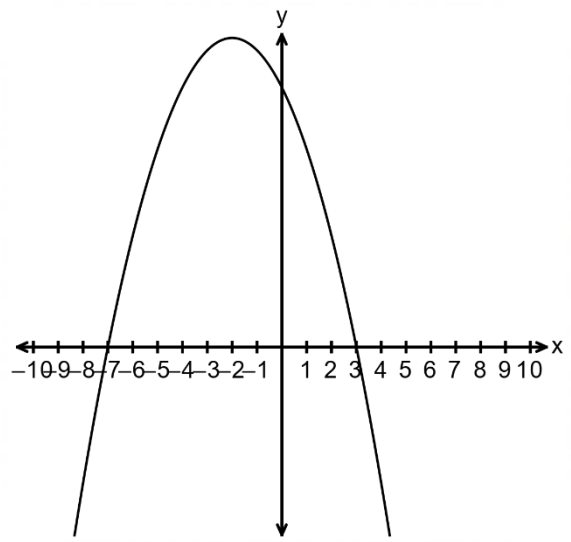
a. $f'(x)<0$ (2)

b. $f''(x)=0$ (1)

c. $xf(x)\leq 0$ (4)

d. $f'(x)f''(x)>0$ (4)

12. The graph of $f'(x)$ is drawn.



a. Determine the x-coordinates of the stationary points of $f(x)$ and justify the nature of the stationary points. (4)

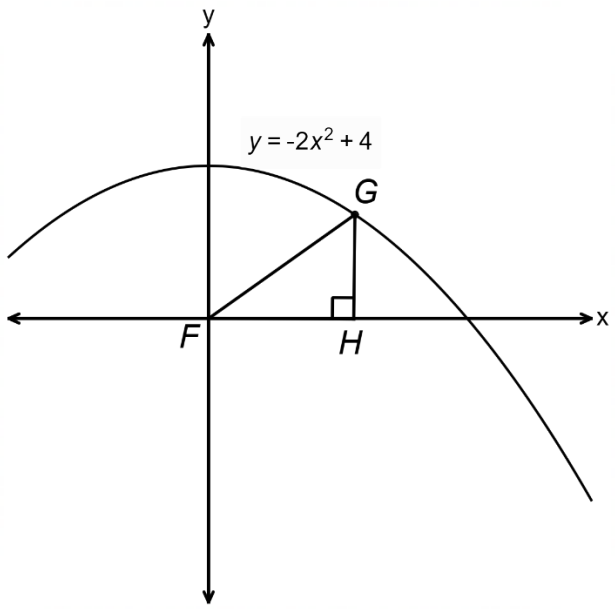
b. For which value(s) of x is $f(x)$ concave down? (1)

13. Sibonelo takes a piece of rope 100 cm long. He cuts it into two pieces and makes a square with the one piece and circle with the other. If the length of the side of the square is x then:

a. Determine an expression for the total area enclosed by the two shapes. (3)

b. Determine x if Sibonelo wants the total area enclosed by the two shapes to be as small as possible. (4)

14. A right angled ΔFGH with $\hat{H} = 90°$ and F at the origin has G on the curve $y = -2x^2 + 4$ as shown. The x-coordinate of G *is positive.*



Determine the x-coordinate of G if ΔFGH is to have maximum area. (6)