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Resource for Day 9 – Trigonometry – Tuesday 31 October

1. Simplify without a calculator:

a.

$$\frac{2\sin(180°-\theta)\cos200°\sin160°}{\cos(\theta-270°)\sin(-40°)}$$

$$\frac{2\sin\theta(-\cos20°)(\sin20°)}{\cos(360°-\theta-270°)(-\sin40°)}$$

$$\frac{2\sin\theta(-\cos20°)(\sin20°)}{\cos(90°-\theta)(-\sin2(20°))}$$

$$\frac{\cancel{2\sin\theta}^{1}(-\cos20°)(\sin20°)}{\cancel{\sin\theta}^{1}(-\sin2(20°)}$$

$$\frac{2(-\cos20°)(\sin20°)}{-2\sin20°\cos20°}$$

$$\frac{\cancel{2}^{1}\left(\cancel{-\cos20°}\right)^{1}\cancel{(\sin20°)}^{1}}{-\cancel{2}^{1}\cancel{(\sin20°)}^{1}\cancel{(\cos20°)}^{1}}$$

$$=-1$$

b.

$$\cos75°$$
$$=\cos(45°+30°)$$
$$=\cos45°\cos30°-\sin45°\sin30°$$
$$=\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\times\frac{1}{2}$$
$$=\frac{\sqrt{6}-\sqrt{2}}{4}$$

c.

$$\frac{\cos(90°-2\theta)}{\cos(-\theta)\sin(90°-\theta)+\sin\theta\sin(-\theta)}$$
$$=\frac{\sin(2\theta)}{\cos\theta\times\cos\theta-\sin\theta\sin\theta}$$
$$=\frac{\sin(2\theta)}{\cos^2\theta-\sin^2\theta}$$
$$=\frac{\sin(2\theta)}{\cos(2\theta)}$$
$$=\tan(2\theta)$$

2. If $\cos 24° = g$ then determine the following in terms of g:

a. $\cos 336°$

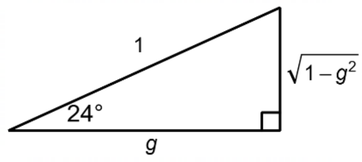
$= \cos 24°$

$= g$

b. $\tan 166°$

$= -\tan 24°$

$= -\dfrac{\sqrt{1-g^2}}{g}$



c. $\sin 138°$

$= \sin(90° + 48°)$

$= \cos 48°$

$= \cos 2(24°)$

$= 2\cos^2 24° - 1$

$= 2g^2 - 1$

d. $\cos 66°$

$= \cos(90° - 24°)$

$= \sin 24°$

$= -\sqrt{1-g^2}$

e. $\cos 192°$

$= -\cos 12°$

now $\cos 2(12°) = 2\cos^2 12° - 1$

$\therefore g = 2\cos^2 12° - 1$

so, $g + 1 = 2\cos^2 12°$

so, $\dfrac{g+1}{2} = \cos^2 12°$

and $\cos 12° = \sqrt{\dfrac{g+1}{2}}$

so, $\cos 192° = -\sqrt{\dfrac{g+1}{2}}$

3. Determine the general solution of the following equations:

a. $2\tan\theta + 1 = -2$

$\tan\theta = -\dfrac{3}{2}$

quadrants 2 and 4

key $\angle = \tan^{-1}\left(\dfrac{3}{2}\right) = 56{,}3$

$\theta = 180° - 56{,}3° + 180k$

$\theta = 123{,}7 + 180k$ with $k \in \mathbb{Z}$

b. $\sin 2\theta \cos 40° - \cos 2\theta \sin 40° = -\dfrac{1}{2}$

$\sin(2\theta - 40°) = -\dfrac{1}{2}$

quadrants 3 and 4

key $\angle = \sin^{-1}\left(\dfrac{1}{2}\right) = 30°$

$2\theta - 40° = 210° + 360k$ *OR*

$2\theta - 40° = 330° + 360k$

$\theta = 125° + 180k$ *OR*

$\theta = 185° + 180k$ both with $k \in \mathbb{Z}$

c. $\tan 2\theta = \tan(\theta + 10°)$

$2\theta = \theta + 10° + 180k$

$\theta = 10° + 180k$ with $k \in \mathbb{Z}$

d. $3\cos^2\theta - 2\sin\theta - 2 = 0$

$3(1 - \sin^2\theta) - 2\sin\theta - 2 = 0$

$3 - 3\sin^2\theta - 2\sin\theta - 2 = 0$

$-3\sin^2\theta - 2\sin\theta + 1 = 0$

$3\sin^2\theta + 2\sin\theta - 1 = 0$

$(3\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{3}$ or $\sin\theta = -1$

quadrants 1 and 2 with

key $\angle = \sin^{-1}\left(\frac{1}{3}\right) = 19{,}5°$

$\theta = 19{,}5° + 360k$ *OR*

$\theta = 160{,}5° + 360k$

OR

$\theta = 270° + 360k$ all with $k \in \mathbb{Z}$

e. $3\cos(2\theta) + 5\sin(2\theta) = 0$

$3\cos(2\theta) = -5\sin(2\theta)$

$-\frac{3}{5} = \tan(2\theta)$

quadrants 2 and 4

key $\angle = \tan^{-1}\left(\frac{3}{5}\right) = 31°$

$2\theta = 180° - 31° + 180k$

$\theta = 74{,}5 + 90k$ with $k \in \mathbb{Z}$

4. Determine the maximum value of $y = 5(\sin\theta\cos 40° + \cos\theta\sin 40°) - 3$

$y = 5(\sin\theta\cos 40° + \cos\theta\sin 40°) - 3$

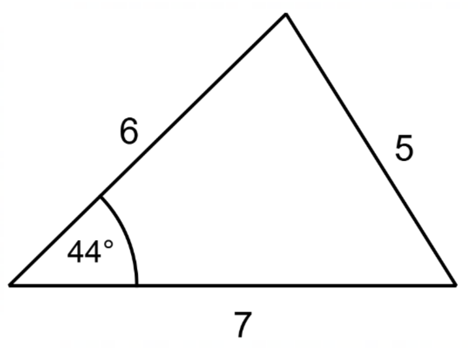
$y = 5\sin(\theta + 40°) - 3$

max value of $\sin(\theta + 40°) = 1$

so max value of y is 2

5. Find the smallest angle in a triangle with sides of 5 cm, 6 cm and 7 cm.

This will be opposite the smallest side!



$\cos\theta = \frac{6^2 + 7^2 - 5^2}{2(6)(7)}$

$\cos\theta = 0{,}71428571$

$\theta = \cos^{-1}(0{,}71428571)$

$\theta = 44{,}4°$

6. Prove that: $\dfrac{\cos 2\theta - \cos\theta}{\sin 2\theta + \sin\theta} = \dfrac{\cos\theta - 1}{\sin\theta}$

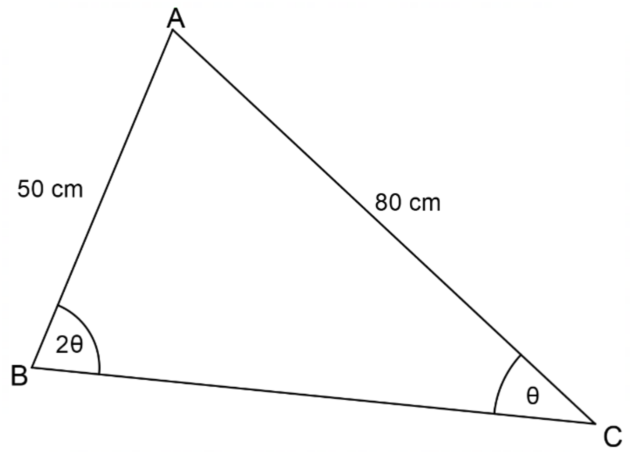
$$LHS = \frac{2\cos^2\theta - 1 - \cos\theta}{2\sin\theta\cos\theta + \sin\theta}$$

$$= \frac{(2\cos\theta + 1)(\cos\theta - 1)}{\sin\theta(2\cos\theta + 1)}$$

$$= \frac{(\cos\theta - 1)}{\sin\theta}$$

$$= RHS$$

7. Find the area of ΔABC



$$\frac{80}{\sin 2\theta} = \frac{50}{\sin\theta}$$

$$\frac{80}{2\sin\theta\cos\theta} = \frac{50}{\sin\theta}$$

$$\frac{40}{\cos\theta} = 50$$

$$40 = 50\cos\theta$$

$$\cos\theta = \frac{4}{5}$$

$$\theta = \cos^{-1}\left(\frac{4}{5}\right) = 36{,}87^\circ$$

$$\hat{A} = 69{,}39$$

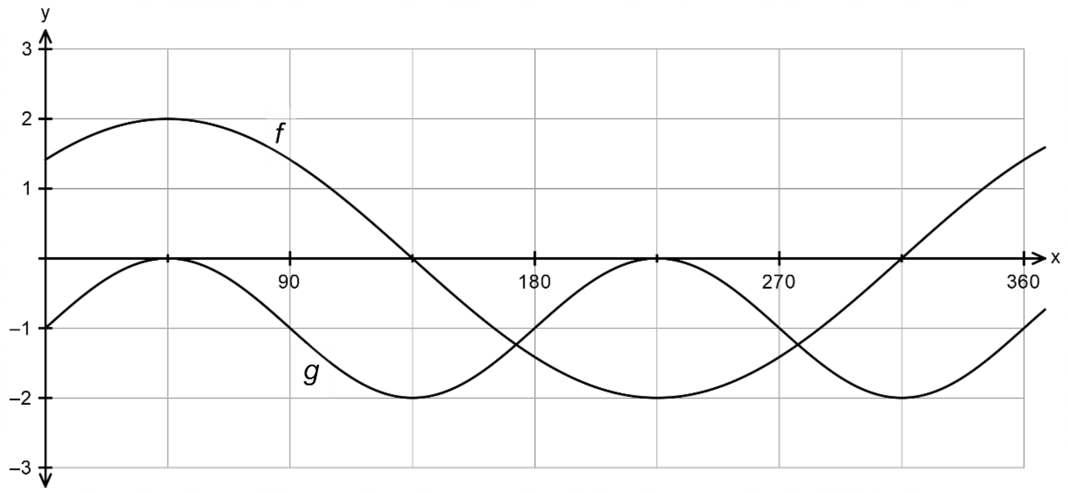
$$Area = \frac{1}{2}(80)(50)\sin 69{,}39^\circ$$

$$Area = 1\,872\text{cm}^2$$

8. Prove that: $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$$
\begin{aligned}
LHS &= \sin(2\theta + \theta) \\
&= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta \\
&= 2\sin\theta\cos\theta\cos\theta + (1 - 2\sin^2\theta)\sin\theta \\
&= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta \\
&= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta \\
&= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta \\
&= 3\sin\theta - 4\sin^3\theta
\end{aligned}
$$

9. Consider the functions *f* and *g* drawn below:

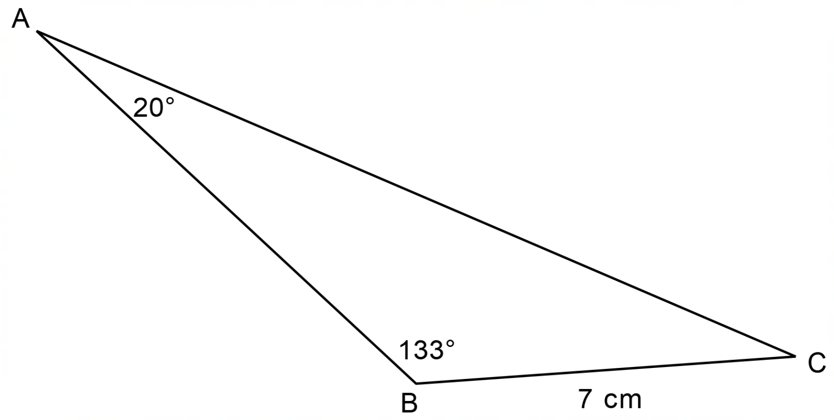


Determine their equations.

$f(x) = 2\cos(x - 45°)$ or $2\sin(x + 45°)$

$g(x) = g(x) = \sin(2x) - 1$

10. Determine the area of ΔABC to one decimal place.



$\hat{C} = 27°$

$\frac{AB}{\sin 27°} = \frac{7}{\sin 20°}$

$AB = \frac{7(\sin 27°)}{\sin 20°}$

$AB = 9.3 \text{ cm}$

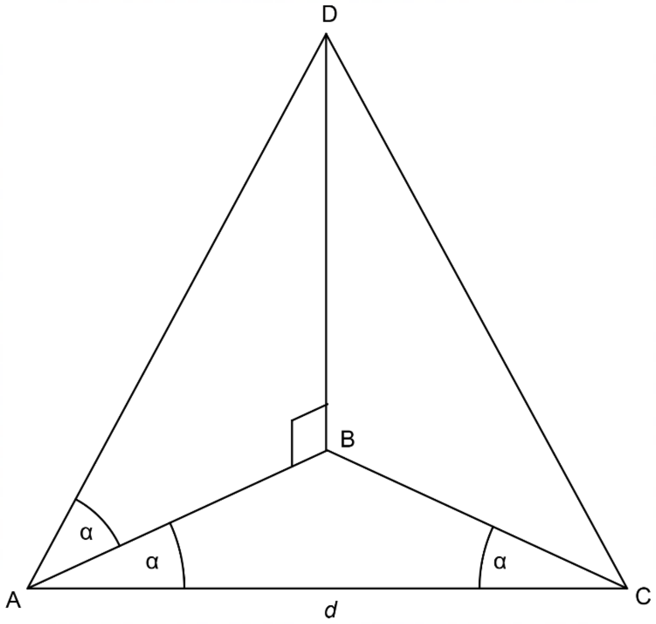
Area $\Delta ABC = \frac{1}{2}(9.29)(7)\sin(133°)$

Area $\Delta ABC = 23.8 \text{ cm}^2$

11. Prove that: $\frac{\sin\theta}{\cos\theta - \sin\theta} + \frac{\sin\theta}{\cos\theta + \sin\theta} = \tan 2\theta$

$$
\begin{aligned}
LHS &= \frac{\sin\theta}{\cos\theta - \sin\theta} + \frac{\sin\theta}{\cos\theta + \sin\theta} \\
&= \frac{\sin\theta(\cos\theta + \sin\theta) + \sin\theta(\cos\theta - \sin\theta)}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)} \\
&= \frac{\sin\theta\cos\theta + \sin^2\theta + \sin\theta\cos\theta - \sin^2\theta}{\cos^2\theta - \sin^2\theta} \\
&= \frac{2\sin\theta\cos\theta}{\cos 2\theta} \\
&= \frac{\sin 2\theta}{\cos 2\theta} \\
&= \tan 2\theta \\
&= RHS
\end{aligned}
$$

12. A vertical tower DB has its foot B in the same horizontal plane as points A and C. A and C are d metres apart. $B\hat{A}C = B\hat{C}A = \alpha$ and the angle of elevation of D from A is α.



Prove that the height of the tower is given by $DB = \dfrac{d\sin\alpha}{2\cos^2\alpha}$

$$\frac{AB}{\sin\alpha} = \frac{d}{\sin(180° - 2\alpha)}$$

$$AB = \frac{d\sin\alpha}{\sin 2\alpha}$$

$$AB = \frac{d\sin\alpha}{2\sin\alpha\cos\alpha}$$

$$AB = \frac{d}{2\cos\alpha}$$

$$\tan\alpha = \frac{DB}{AB}$$

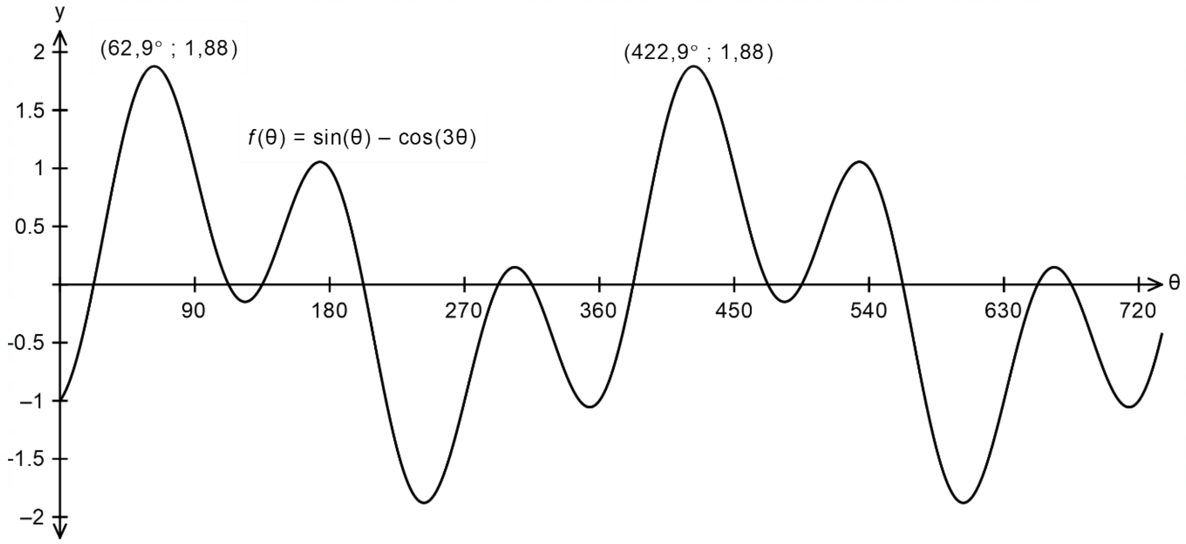
$$\tan\alpha = \frac{DB}{AB}$$

$$DB = AB\tan\alpha$$

$$DB = \frac{d}{2\cos\alpha} \times \frac{\sin\alpha}{\cos\alpha}$$

$$DB = \frac{d\sin\alpha}{2\cos^2\alpha}$$

13. The function $f(\theta) = \sin(\theta) - \cos(3\theta)$ is drawn below for $\theta \in [0°;720°]$



a) Give the period of f

360°

b) Determine a general solution for the x-intercepts of f.

$\sin(\theta) - \cos(3\theta) = 0$

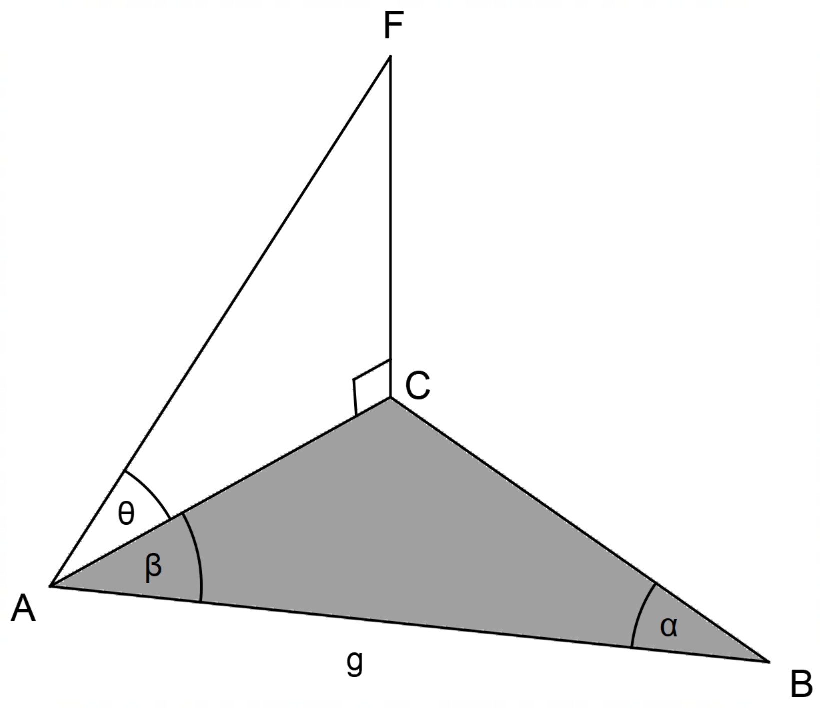
$\sin(\theta) = \cos(3\theta)$

$\sin(\theta) = \sin(90° - 3\theta)$

$\theta = 90° - 3\theta + 360k$ or $\theta = 180° - (90° - 3\theta) + 360k$ both with $k \in \mathbb{Z}$

$\theta = 22.5° + 90k$ or $\theta = -45° + 180k$ both with $k \in \mathbb{Z}$

14. In the picture A, B and C are in the same horizontal plane. C is the foot of a **vertical** tower, FC. The distance between A and B is *g*. The angle of elevation of F from A is θ.
$C\hat{A}B = \beta$ and $C\hat{B}A = \alpha$.



Show that $FC = \dfrac{g\sin\alpha\tan\theta}{\sin(\alpha+\beta)}$

$$\frac{g}{\sin(180° - (\alpha+\beta))} = \frac{AC}{\sin\alpha}$$

$$\therefore AC\sin(\alpha+\beta) = g\sin\alpha$$

$$\therefore AC = \frac{g\sin\alpha}{\sin(\alpha+\beta)}$$

$$\tan\theta = \frac{FC}{AC}$$

$$\therefore FC = AC\tan\theta$$

$$\therefore FC = \frac{g\sin\alpha\tan\theta}{\sin(\alpha+\beta)}$$